The Status Quo Problem: Ruling out Welfare

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Abstract

We consider whether welfare under Social Security explains the continued popularity of the system. Heterogeneous ability/income which is correlated with longevity is incorporated into a large scale general equilibrium overlapping generations economy. This model is used to measure welfare under both Social Security and No Social Security, in a variety of alternative settings. Compensating variations and willingness-to-pay are analyzed by type of worker, and shown to not be an obstacle to reform. An exception to the rule is found with the Hybrid Reform, in which welfare increases in a temptation framework.

1 Introduction

For over thirty years, economists have studied the effect of Social Security on capital formation. Consistently, studies that employed a life-cycle consumer model have shown that agents reduce saving in the economy with Social Security. Lower savings leads to lower levels of capital, output, consumption and welfare in a general equilibrium steady state. [1] A notable exception to the conclusion that Social Security reduces steady state welfare is when the decision maker is the head of a multi-generation family unit (Fuster 1999). In this dynastic setting, Social Security, which performs the type of intergenerational transfers a dynastic decision maker would affect in the absence of Social Security, is effectively nullified by the actions of the decision maker. Hence in a dynastic setup, there is little difference in the steady state of the economy with or without Social Security. Whether a dynastic setup or a life-cycle model is more valid is not known. What is clear is that if a life cycle model is valid, then Social Security reduces steady state capital, output and welfare, and if a dynastic setup is valid, then Social Security is cancelled out by the multi-generation household decision maker.

It is natural to wonder then, why Social Security remains such a popular public program. Perhaps it is simply a result of the schedule of taxes, which

[1] This has been shown in two period models, in overlapping generation models with standard preferences, with time inconsistent preferences and with temptation preferences.

are back loaded to increase in the future when the heaviest burden of retirements will occur. Perhaps people do not appreciate the future costs yet to be borne. But assuming this is not the case, and that people are fully aware of the rising tax burden ahead, then what accounts for its continued popularity? Why is it so difficult to reform the system to one that leads to higher steady state outcomes? This is the "status quo problem". The implication is that the short term pain to change the system offsets the longer term gain (to another generation) and thereby blocks reform. This paper looks at the status quo problem in terms of the welfare of the present worker. The question at the heart of the paper is whether or not the present system gets even close to enhancing welfare enough that todays workers are willing to pay for the true costs of Social Security. Experiments will include bequests in the utility of workers, a progressive (kinked) benenfit formula to benefit low income workers, increasing the age at which benefits start to be received, and alternative elasticities of intertemporal substitution. We will also consider whether workers with temptation preferences, (i.e., self-control issues), would achieve higher individual welfare with Social Security than without it. In each case we consider the effect on capital formation and welfare in a model economy with Social Security and with No Social Security.

It seems an odd question, given that workers have happily paid into the Social Security system for over 70 years. Yet the history of Social Security is one of benefits expanding faster than the costs to workers, so that there have been first generation "windfalls" to workers already in the system for the period up to 1977. Since the overhaul of the system in connection with the 1983 Social Security reform commission, benefit expansion has stopped, and the changes to Social Security in 1983 and in 1996 have been in the direction of reduced benefits and higher taxes. This is clearly in the future of the system again as actuarial imbalances are corrected. Is there evidence that enough welfare is generated in the system to make these coming tax increases (or benefit reductions) politically palitable?

The findings of the paper can be summarized as follows:

1. Low income workers are willing-to-pay more for Social Security benefits than high income workers are willing-to-pay, but there is not a large enough difference (under a plausible parameterization of the model) to account for the popularity of Social Security.

2. Diminishing marginal utility means that improved individual welfare emenating from reduced uncertainty in retirement income under Social Security does not increase in line with an increasing level of Social Security benefits.

3. Starting the pension benefits at a later age increases individual welfare relative to the cost of of the pension.

4. Including bequests in utility does not make a significant difference (assuming a "joy of giving" specification for bequests).

5. Elasticity of intertemptoral substitution affects saving decisions in the Social Security economy steady state in the opposite direction than in the No Social Security steady state.

6. In a framework with temptation, the welfare loss due to Social Security

is slightly smaller than in a framework without temptaion. The difference in welfare between high income and low income workers is noticabley reduced from the difference in welfare in the framework without temptation.

7. In a framework with temptation, a social security mechanism which includes an effective commitment device could improve individual welfare. Moreover, it appears possible that an effective commitment device could improve welfare enough that agents would be willing to pay the required tax for the public pension.

The paper is organized as follows. We start by considering a simple two period model of Social Security, to gain intuition into the effect of our experiments. We consider related research in Section 2. The model and its calibration are presented in Sections 3 and 4. Results are in Section 5. After some closing remarks, the Appendix contains details of the results of the experiments.

2 Two Period Model of Social Security

To gain insight into what to expect from a large overlapping generations model of Social Security, we start our analysis with a simple two period model in which factor prices are givien. An agent lives two periods, with the probability of surviving to period two denoted by ψ. The agent receives a bequest (B) and a wage (w) in period one. The agent pays a tax on wages at rate τ to fund a social security benefit in period two. The size of the social security benefit is $\theta \cdot w$. Savings earn a gross return R. The agent has a time preference discount factor of β. The agent must choose consumption in each of the two periods $\{c_1, c_2\}$, in such a way as to maximize utility. We assume isoelastic utility. The agent's problem is:

$$\max u(c_1) + \beta\psi u(c_2)$$

$$\text{subject to:} \qquad u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

$$c_2 = [B + w(1-\tau) - c_1] + \varphi w \tag{1}$$

$$\tau w = \psi\theta w \tag{2}$$

The first order condition is:

$$\frac{c_2}{c_1} = (\beta\psi R)^{\frac{1}{\gamma}}$$

Solving for c_1 (and hence for c_2 also):

$$c_1 = \frac{BR + w[(1-\tau)R + \frac{\tau}{\psi}]}{R + (\beta\psi R)^{\frac{1}{\gamma}}}$$

With this result we examine the effect on saving of several experiments. We start with an experiment in which social security benefits are received at an older age. In terms of the two period model, this is as if the probability of survival is reduced. We look to the calculus for help and consider $\frac{\partial c_1}{\partial \psi}$.

$$\frac{\partial c_1}{\partial \psi} = \{BR + w[(1-\tau)R + \frac{\tau}{\psi}]\}\{\frac{-1}{[R + (\beta\psi R)^{\frac{1}{\gamma}}]^2}\}\{\beta R\frac{1}{\gamma}(\beta\psi R)^{\frac{1}{\gamma}}\} + ... \\ \frac{1}{R + (\beta\psi R)^{\frac{1}{\gamma}}}[\frac{-wR}{\psi^2}]$$

We find that the derivative is always negative, so that as the probability of survival (ψ) decreases, consumption in period one increases, and savings decrease. Hardly surprising, but it gives some insight before we consider a more complex model.

Suppose we increase the social security benefit in period two. What effect does that have on savings and consumption? In the model, increasing benefits is equivalent to increasing τ, so we look to the derivative with respect to τ.

$$\frac{\partial c_1}{\partial \tau} = \frac{1}{R + (\beta\psi R)^{\frac{1}{\gamma}}}[\frac{w}{\psi}] - \frac{1}{R + (\beta\psi R)^{\frac{1}{\gamma}}}[wR]$$

In this case, $\frac{\partial c_1}{\partial \tau}$ is positive if $\frac{1}{\psi} > R$, and $\frac{\partial c_1}{\partial \tau}$ is negative if $\frac{1}{\psi} < R$. (The "return on survivorship" is $\frac{1}{\psi}$, while the return on savings is R.) Thus, if the age at which the benefits are received is old enough so that the survivorship return is greater than the return on savings, i.e. $\frac{1}{\psi} > R$, then $\frac{\partial c_1}{\partial \tau}$ will be positive, and an increase in taxes (and benefits) will also result in an increase in c_1, and a decrease in savings. This is a happy result, whereby increased taxes lead to greater consumption in period one and in period two. But if the return on survivorship is less than the return on savings, $\frac{1}{\psi} < R$, then $\frac{\partial c_1}{\partial \tau}$ will be negative, and an increase in taxes will reduce c_1.

Finally, we are interested in the effect that different degrees of risk aversion might have in the model. Since returns are certain in this model, the risk aversion parameter γ is more appropriately interpreted as the inverse of the constant elasticity of interptemporal substitution. When γ is large, the inverse is small, and the agent is more willing to shift consumption between periods to improve utility. But when γ is small and the inverse is large, then the agent is less willing to shift consumption between periods. We consider again the derivative, this time with respect to $\frac{1}{\gamma}$, the elasticity of intertemporal substitution.

$$\frac{\partial c_1}{\partial \frac{1}{\gamma}} = \{BR + w[(1-\tau)R + \frac{\tau}{\psi}]\}\{\frac{-1}{[R + (\beta\psi R)^{\frac{1}{\gamma}}]^2}\}\{(\beta\psi R)^{\frac{1}{\gamma}}\log(\beta\psi R)\}$$

In this case, the key to the sign of the derivative is the term $\log(\beta\psi R)$. If $\log(\beta\psi R) < 0$, then $\frac{\partial c_1}{\partial \frac{1}{\gamma}} > 0$, and if $\log(\beta\psi R) > 0$, then $\frac{\partial c_1}{\partial \frac{1}{\gamma}} < 0$. Assuming that $0 < \beta < 1$, we have $\log(\beta\psi R) > 0 \iff \beta\psi R > 1 \iff R > \frac{1}{\beta\psi}$. If $R > \frac{1}{\beta\psi}$, (which would be the case if the age at which benefits are received is not too old), then $\frac{\partial c_1}{\partial \frac{1}{\gamma}} < 0$. Thus as $\frac{1}{\gamma}$ increases (becomes more inelastic),

c_1 decreases and savings increases. But if $R < \frac{1}{\beta\psi}$ (which could happen if the age at which benefits are received is old enough), then $\frac{\partial c_1}{\partial \frac{1}{\gamma}} > 0$, and then consumption increases as $\frac{1}{\gamma}$ increases, and savings decreases. This insight will be helpful when considering the complex interaction of mortality, interest and preferences with respect to intertemporal choices.

Related Literature.

Yet to be done.

3 The Overlapping Generations Model

The OLG model used in this paper is essentially the same as the one used in an earlier paper (Boronow 2007), with a couple noteworthy changes. Most importantly, this paper includes bequests in the utlity function as an experimental option. Also, the model includes a kinked benefit formula matching the formula used by the Social Security Administration, as an alternative experiment to the simple average replacement ratio used in the earlier paper. Finally, while the earlier paper studied the effect of temptation preferences on steady state outcomes of model economies, this paper uses standard preferences as it studies the changes to welfare due to the impact of experimental changes to the model.[2] We come back to temptation preferences later as another experimental framework for analysis of welfare.

3.1 Demographics

Time is discrete, and each period represents one year. Age 1 corresponds to real age 21. The oldest possible age is age J , where $J = 85$ (real age 105). We assume that death is certain thereafter.

There are two types of agents indexed by z, where $z \in \{1, 2\}$. An agent's type is revealed at birth and this determines lifetime ability, which can be either high ($z = 1$) or low ($z = 2$). The realization of ability follows a first-order Markov process with transition matrix Π:

$$\begin{aligned} \Pi(z, z') &= [\pi_{ij}]; i, j \in \{1, 2\} \\ \pi_{ij} &= \Pr(z' = j | z = i). \end{aligned}$$

where z is the ability type of the parent and z' is the ability type of the child. It is assumed that the transition probabilities, π_{ij}, are such that there is a resulting stationary distribution of ability types, Λ, where $\lambda(z) \in \Lambda$, and $\lambda(1) + \lambda(2) = 1$.

The ability type determines the endowment of efficiency units an agent receives. In a given period, the cross-sectional labor efficiency $\varepsilon_j(z)$ is indexed by ability type z and age j. Without loss of generality, we assume throughout this paper that the rate of technological growth is zero. Under this assumption,

[2] One further technical difference is that this model is solved on a uniform grid with 5000 points. The earlier paper used a non-uniform grid with 7401 points.

the longitudinal efficiency units of a particular agent equal the cross-sectional efficiency factors, $\varepsilon_j^l(z) = \varepsilon_j(z)$.

Agents have uncertain lifetimes. Survival probabilities are correlated with ability type, so that high ability agents have longer expected lifetimes than low ability agents. Thus survival rates are indexed by age and type. The probability that an agent age j and ability type z survives to age $j+1$ is denoted by $\psi_j(z)$. The probability that an agent age j and ability type z survives to age $j+t$ is denoted by $\Psi_{j,t}(z)$, where:

$$\begin{aligned} \Psi_{j,t}(z) &= 1, \quad \text{if } t = 0 \\ \Psi_{j,t}(z) &= \Pi_{s=1}^{t} \psi_{j+s-1}(z), \ \text{if } t > 0. \end{aligned}$$

Like much of the social security literature, this paper analyzes the steady states of a stationary population distribution, with time invariant cohort shares. Let ρ be the assumed constant rate of growth in population. Then, the cohort share of a new agent of type 1 relative to a new agent of type 2 is equal to $\frac{\lambda(1)}{\lambda(2)}$. That is, the size of the newborn type cohorts, relative to each other is determined by the Markov process stationary distribution of types. Thereafter, relative cohort shares are a result of the population growth rate and survival probabilities. Letting $\mu_j(z)$ denote the cohort share for an agent of age j and ability type z, and letting N denote the total population when the newborn cohort is indexed to one (1), for newborns ($j = 1$):

$$\begin{aligned} \mu_1(z) &= \lambda(z) \cdot [1/N], \text{ where} \\ N &= \sum_{z=1}^{2} \sum_{t=0}^{J} \lambda(z) \cdot (1+\rho)^{-t} \cdot \Psi_{1,t}(z). \end{aligned} \tag{1}$$

Each new cohort is $(1+\rho)$ times as large as the preceding cohort, and each cohort survives to the next period according to the corresponding age and ability type, $\psi_j(z)$. Thus for $j = 1, 2, ...J-1$:

$$\mu_{j+1}(z) = \mu_j(z) \cdot \frac{\psi_j(z)}{1+\rho}. \tag{2}$$

Finally, the sum of all cohorts must equal 100% so that,

$$\sum_{z=1}^{2} \sum_{j=1}^{J} \mu_j(z) = 1. \tag{3}$$

Given the Markov process, survival rates, and population growth rate, $\Pi, \psi_j(z)$ and ρ, respectively, the above relationships uniquely determine the time invariant cohort shares, $\{\mu_j(z)\}$.

3.2 Technology and Factor Prices

There is a single good in the economy, produced by one or more firms using a constant returns to scale Cobb-Douglas production function:

$$Y = AK^{1-\alpha} \cdot L^{\alpha}, \text{ where } \alpha \in (0,1).$$

Total factor productivity A is normalized to 1. The labor share is α and K and L are aggregate capital and labor supplied as inputs. Capital is assumed to depreciate at the constant rate δ. Therefore, in a competitive equilibrium, we get factor prices for capital and labor:

$$\begin{aligned} r &= (1-\alpha) \cdot K^{-\alpha} \cdot L^{\alpha} - \delta \\ w &= \alpha \cdot K^{1-\alpha} \cdot L^{\alpha-1} \end{aligned} \tag{4}$$

K represents the aggregate asset holdings over the population in a given period. The size of L is determined by the workers up to retirement age j^*. Workers are assumed to supply labor inelastically to age j^*, and do not work thereafter. The actual supply of efficient labor depends on the ability type of agents in the working age population.

$$L = \sum_{z=1}^{2} \sum_{j=1}^{j^*-1} \varepsilon_j(z) \cdot \mu_j(z). \tag{5}$$

3.3 Government Policy and Social Security

This paper will analyze two model economies that differ in their approach to Government Policy and Social Security. There is a No Social Security economy (NSSE), and a Social Security economy (SSE).

3.3.1 No Social Security Model Economy

Government policy in the NSSE model is straightforward. There is no social security tax, and there are no social security benefits. Each worker must prepare for retirement income by saving during their working years, to build up a retirement nest egg.

3.3.2 Social Security Model Economy

In the SSE model, there is a social security program that provides a public pension to retirees. Average lifetime earnings for a worker of ability type z, denoted by $w^{SS}(z)$, is given by:

$$w^{SS}(z) = \frac{1}{j^* - 1} \cdot \sum_{j=1}^{j^*-1} w\varepsilon_j(z) \tag{6}$$

The social security benefit, denoted by $b^{SS}(z)$, is determined as a percentage of average lifetime earnings, where the percentage decreases as the average lifetime earnings increase. Let $wbar$ denote the average lifetime earnings over all types of workers. Then for low income workers ($z = 2$),

$$b^{SS}(2) = .9 * (.2 * wbar) + .33 * (w^{SS}(2) - .2 * wbar)$$

For high income workers ($z = 1$),

$$b^{SS}(1) = .9 * (.2 * wbar) + .33 * ((1.25 - .2) * wbar) + .15 * (w^{SS}(1) - 1.25 * wbar)$$

The set of breakpoints (20% and 125%), together with the factors .9, .33, .15, are collectively referred to as policy choice θ.

In some cases, we will want to produce results using a social security benefit which is simply $b^{SS}(z) = .4 * w^{SS}(z)$. Here we set $\theta = .4$. It is noted wherever the flat benefit formula is used as model economy SSE-40.

The role of the government is to collect a tax on labor income to exactly provide the social security pension to retirees. The necessary tax rate, τ_{SS}, in this pay-as-you-go model is:

$$\tau_{SS} = \frac{\sum_{z=1}^{2} \sum_{j=j^*}^{J} b^{SS}(z) \cdot \mu_j(z)}{w \cdot \{\sum_{z=1}^{2} \sum_{j=1}^{j^*-1} \varepsilon_j(z) \cdot \mu_j(z)\}} \tag{7}$$

The numerator is the total benefit paid under social security and the denominator is the total wage base over which the tax is applied in a given period.

3.4 Constraints and Bequests

During the working years, the agent receives after-tax labor income based on their age/ability profile of labor efficiency. Upon retirement in the SSE model, the agent receives social security benefits. In the NSSE model, there are no social security benefits.

Each period, the agent must choose the amount of consumption and the amount of voluntary saving. Savings earn the rate of return on capital r. Agents are subject to a no borrowing constraint.

Because lifetimes are uncertain, many agents will die with positive amounts of assets (aka accidental bequests). It is not uncommon for researchers to assume that these unintended bequests are confiscated and redistributed equally (per effective worker) to the survivors. This paper modifies that assumption somewhat. In this paper, accidental bequests are redistributed to surviving agents, in such a way that each type of agent receives an equal share based on the expected bequest of that agent, given their ability type. The amount of the bequest distributed to agents of ability type z is denoted by $\xi(z)$.

We can now describe the budget constraint faced by an agent of age j and ability type z, which is given by:

$$c_j(z) + a_{j+1}(z) = [a_j(z) + \xi(z)] \cdot (1 + r) + Q_j(z) \tag{12}$$

The left hand side of the equation is the allocation of that period's wealth to consumption and savings, while the right hand side is the total of the resources available from prior savings and returns, bequests, wages and benefits (if any) from the social insurance mechanism. In all economies, newborn agents enter with zero assets ($a_1(z) = 0$). In the final period, optimality requires that $a_{J+1}(z) = \omega^{\frac{1}{\gamma}} c_J$. (The parameter ω is the strength of the bequest motive, as defined later.) When bequests are not in the utility function, the optimality condition becomes $a_{J+1}(z) = 0$. Further, for the three model economies, Q_j is defined as follows. In the No Social Security economy (NSSE):

$$\begin{aligned} Q_j(z) &= w \cdot \varepsilon_j(z) \ \text{ for } j < j^* \\ Q_j(z) &= 0 \ \text{ for } j^* \leq j \end{aligned}$$

In the Social Security economy (SSE):

$$\begin{aligned} Q_j(z) &= w \cdot \varepsilon_j(z) \cdot (1 - \tau_{SS}) \text{ for } j < j^* \\ Q_j(z) &= b^{SS}(z) \text{ for } j^* \leq j \end{aligned}$$

In all economies, households face a borrowing constraint:

$$a_j(z) \geq 0, \ \forall \ j$$

The bequest $\xi(z)$ is defined below.

3.4.1 Expected Bequests

Since ability type determines labor earnings, then the average size of an accidental bequest differs by type. It is reasonable to assume that children receive the accidental bequest left by the parent. The problem is how to allocate accidental bequests to agents of type 1 and type 2 so that the allocation is consistent with a presumption that the bequest stay in the family. To do this, let $Beq(z)$ denote the average bequest of agents of type z that die in a given period:

$$Beq(z) = \frac{\sum_{j=1}^{J} \{a_j(z) \cdot \mu_j(z) \cdot (1 - \psi_j(z))\}}{\sum_{j=1}^{J} \mu_j(z) \cdot (1 - \psi_j(z))} \tag{13}$$

The numerator is the sum of assets owned by the type z agents who die in a given period, while the denominator is the number of such agents.

Recall that $\pi_{ij} \in \Pi$ is the probability that a parent of type i has a child of type j, and that $\lambda(z)$ is the probability that a newborn is type z. It turns out that the probability that a child of type z has a parent of a given type produces exactly the same Markov probability matrix Π. We use this fact to construct the ratio of the conditional expected bequest for agents of type z to the overall average bequest to agents of all types. We then use that ratio to allocate accidental bequests as follows:

$$\xi(z) = \frac{\pi_{z1} \cdot Beq(1) + \pi_{z2} \cdot Beq(2)}{\sum_{z=1}^{2} \lambda(z) \cdot (\pi_{z1} \cdot Beq(1) + \pi_{z2} \cdot Beq(2))} \cdot Beq \tag{14}$$

$$\text{where } Beq = \sum_{z=1}^{2} \sum_{j=1}^{J} \{a_j(z) \cdot \mu_j(z) \cdot (1 - \psi_j(z))\}$$

Despite the extra complexity of this approach, there is a purpose in the specification of accidental bequests in such a way that it takes account of the likely ability type of the parent of the agent. That purpose is to build into the model, at least to this degree, the intergenerational effect of bequests under the alternative security regimes, when mortality is correlated with ability/income. It is thought that with bequests in the utility function, intergenerational effects of Social Security will perhaps be more relevent.

3.5 Preferences and Individual Optimization Problem

This model includes bequests in the utility, using a simple form with one additional parameter to indicate the strength of the "joy of giving". We otherwise use the same isoelastic form of utility for bequests that we use for consumption.

To streamline notation, let a denote $a_j(z)$, where age and type are defined by the context of the usage. Likewise, we use the same notational shortcut for c, Q and ξ. Following notational convention, the prime symbol denotes the next period value.

Standard preferences are defined over a lifetime sequence of consumption $\{c_j(z)\}_{j=1}^{J}$. In the case where utility is over consumption (consumption in utility, CIU), the individual agent's objective for an agent age j is to maximize expected discounted lifetime utility:

$$U_j = \sum_{t=0}^{J-j} \beta^t [\Psi_{j,t}(z)] \cdot u(c_{j+t}(z)) \tag{15}$$

where $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, γ is the constant coefficient of relative risk aversion and β is the discount factor. The term in brackets is the probability of survival to age j for an agent of type z.

For the case where consumption and bequests are in the utility function (CBIU), the expected discounted lifetime utility is:

$$U_j = \sum_{t=0}^{J-j} \beta^t [\Psi_{j,t}(z)] \cdot u(c_{j+t}(z), a'_{j+t}(z))$$

$$\begin{aligned} where \;\; u(c) &= \frac{c^{1-\gamma}}{1-\gamma} + (1-\psi)\omega\frac{a'^{1-\gamma}}{1-\gamma} \\ a' &= (a+\xi)(1+r) + Q - c \end{aligned}$$

As noted above, the parameter ω represents the strength of the bequest motive.

An agent of age j and type z starts a given period with initial asset holdings a. The individual's dynamic problem is to choose how much to consume now c, and how much to save for future consumption a', in order to maximize the Bellman equation:

$$W_j(a) = \max_{c,a'}\{u(c,a') + \beta\psi_j(z)W_{j+1}(a')\} \tag{19}$$

subject to the budget constraint, the borrowing constraint, the initial and optimality conditions, and taking the factor prices as given.

3.6 The Steady State Equilibrium

Let $D = \{d_1, d_2, ..., d_m\}$ represent the discrete set of values that asset holdings are permitted to take. The feasible set for an age j agent of type z and asset holdings a is denoted by $\Omega(j,a,z)$. The possible choices for a satisfy $a' \in \Omega(j,a,z)$, $a' \geq 0$, and the budget constraints.

A steady state equilibrium for a set of policy parameters $\{\theta, \tau_{SS}\}$ is a collection of value functions $\{W_j(a)\}$; decision rules $R^c_{a,j,z} : D \times \{1,2,...,J\} \times \{1,2\} \rightarrow \mathbb{R}_+$ and $R^{a'}_{a,j,z} : D \times \{1,2,...,J\} \times \{1,2\} \rightarrow D$; a stationary distribution of types of newborns, $\{\lambda(1), \lambda(2)\}$; a time invariant distribution of agents by type, $\{\mu_j(z) | \forall j \in \{1,2,...J\}, \forall z \in \{1,2\}\}$; a set of prices for capital and labor $\{r, w\}$; and a set of lump sum transfers of accidental bequests to agents $\{\xi(z)\}$; such that

1. Given factor prices, government policy and the lump sum transfers, the decision rules solve the individual optimization problem.
2. Factor prices solve the optimization problem of the firm
3. Markets clear, implying that:

$$K = \sum_{z=1}^{2}\sum_{j=1}^{J}[a_j(z) + \xi(z)] \cdot \mu_j(z) \tag{21}$$

$$L = \sum_{z=1}^{2}\sum_{j=1}^{j^*-1}[\varepsilon_j(z) \cdot \mu_j(z)] \tag{22}$$

$$Y = \sum_{z=1}^{2}\sum_{j=1}^{J}\mu_j(z) \cdot [c_j(z) + a_{j+1}(z) - (1-\delta) \cdot (a_j(z) + \xi(z)] \tag{23}$$

Thus, aggregate capital is the sum of individual asset holdings, aggregate labor is the sum of effective workers and output equals aggregate consumption plus the increase in aggregate capital.

4. The invariant population distribution conditions are satisfied.

5. Government pensions are fully financed by the labor tax.

6. The lump sum transfers satisfy:

$$\sum_{z=1}^{2}\sum_{j=1}^{J}\xi(z)\cdot\mu_j(z)=\sum_{z=1}^{2}\sum_{j=1}^{J}\{a_j(z)\cdot\mu_j(z)\cdot(1-\psi_j(z))\} \tag{24}$$

4 Calibration and Solution Method

4.1 Calibration

4.1.1 Demographic and Labor Parameters

The model is calibrated following some of the previous literature. Each period represents one year. The probabilities that make up the transition matrix Π are taken from the calibration used by Fuster, Imrohoroğlu and Imrohoroğlu (2002). As they explained, the values of Π were chosen to match 1991 Bureau of the Census data on the proportion of college graduates in the work force, and to match an observed correlation between the permanent component of income of parents and children, based on estimates by Zimmerman (1992) and Solon (1992). Thus:

$$\Pi=\begin{bmatrix}\pi(1,1) & \pi(1,2)\\ \pi(2,1) & \pi(2,2)\end{bmatrix}=\begin{bmatrix}.57 & .43\\ .17 & .83\end{bmatrix}$$

This transition matrix produces a stationary distribution of types for newborns $\Lambda=[\lambda(1)\ \lambda(2)]=[.2833\ .7167]$. Therefore, the proportion of newborns with high ability is 28.33% and the proportion of low ability is 71.67%.

Mortality is assumed to be different between high ability agents and low ability agents. The survival rates are developed in three steps. First, the study begins with the same mortality rates used by Imrohoroğlu, Imrohoroğlu and Joines (1999) in their 65 period model, which is based on mortality rates from the Social Security Administration. The present paper extends the mortality rates 20 more periods, again based on mortality rates from the Social Security Administration. The aggregate mortality rates are then split into two sets of mortality rates; one for the high ability workers and one for the low ability workers. The method used to split aggregate mortality rates by type is explained in Boronow (2007).

The model also splits the aggregate labor efficiency factors into two sets, one for each type. The method used to split aggregate labor efficiencey rates by type is also explained in Boronow (2007). Labor is supplied inelastically to retirement age, which is fixed at $j^*=45$, corresponding to real age 65. Thereafter, $\bar{\varepsilon}_j=0$.

The population growth rate ρ is assumed to be a constant and equal to 1.2%, using the calibration by Fuster, Imrohoroğlu and Imrohoroğlu (2002). This corresponds to the average annual population growth rate of the United States over a fifty year period.

4.1.2 Technology Parameters

The model uses a Cobb-Douglas production function with constant returns to scale, $Y = AK^{1-\alpha}L^{\alpha}$. Total factor productivity A is normalized to one. The labor's share α is set to 0.64. These values are often used in a simple model, as they approximate the observed patterns in the US over a long period. The value of L is determined by the demographic assumptions.

Depreciation is set at a constant rate of 6.9%, following Imrohoroğlu, Imrohoroğlu and Joines (1999), in which they calculated their technology parameters based on annual US data since 1954. The rate of exogenous technological growth is set to zero.

4.1.3 Government Policy and Social Security

For flat benefit formula, the replacement rate θ that is used to model social security is 40%, roughly comparable to the average level of benefits of Social Security. Experiments with this benefit formula are identified as SSE-40. For the kinked benefits formula, θ represents the collection of breakpoints and factors used by the Social Security administration, as discussed earlier. Experiments with this benefit formula are identified as SSE. The default age at which benefits start, j^b, is 45 (corresponding to real age 65) for the Social Security economy, except in experiments where it is noted that the benefit starting age is increased.

4.1.4 Preferences

The parameters that specify standard preferences in the model are the coefficient of relative risk aversion, γ, and the time preference discount rate, β. The coefficient of relative risk aversion, γ, is usually set to 2 (except in those cases where we experiment with variations on γ), and the time preference discount rate, β, is set to .978. These values match the values used by Imrohoroğlu, Imrohoroğlu and Joines (1999), which were chosen to produce a capital intensity ratio close to 2.5, which is the empirical average in the US since 1954, according to their analysis. In this paper, under the SSE model, this parameterization produces a capital intensity of 2.9. While higher than the Imrohoroğlu et al model result, it is closer to the wealth output ratios reported by Cooley and Prescott (1995) and Laitner (1992), which were 3.32 and 3.15 respectively.

4.1.5 Summary of Calibration and Resulting Tax Rates

Table 1 presents a summary of the calibration parameters.

Table 1. Calibration Parameters

Demographics		
Population growth rate	ρ	.012
Maximum survival age	J	105 (model period 85)
Retirement age	j^*	65 (model period 45)
Conditional survival probabilities	$\bar{\psi}(j)$	based on SSA Tables
Low ability mortality factor		112.6%
Efficiency profile	$\bar{\varepsilon}(j)$	Hansen (1993)
High ability efficiency factor		150%
Production		
Labor share	α	.64
Total factor productivity	A	1
Depreciation	δ	.069
Government Policy		
Benefits start age	j^*	65 (except where noted)
Replacement Rate Parameters	θ	40% or $\{.2, 1.25, 90\%, 33\%, 15\%\}$
Preferences		
Relative risk aversion	γ	2 (except where noted)
Strength of "joy of giving"	ω	$\{0, 2\}$ as noted
Discount rate	β	.978

One of the distinguishing characteristics of the alternative economic models is the tax rate which is needed to finance their respective social insurance programs. While technically a result of the model, not a calibration of the model, they are nonetheless the result of the calibration of invariant demographics and government policy (benefit design), not economic behavior. Therefore they are shown below.

Table 2. Social InsuranceTax Rates

Benefit Age	NSSE	SSE (flat 40%)	SSE
65	0	8.77%	9.32%
70	0	6.01%	NA
75	0	3.78%	NA
80	0	2.11%	NA

4.2 Solution Method

The model is solved using a recursive method applied on a discretized state space. The solution we seek is a steady state of the economy. Starting from an initial guess as to the value of aggregate capital, K, and a guess as to the value of aggregate bequests, B, the solution algorithm computes optimal saving and consumption decisions for all agents in the invariant population distribution for a given period. New aggregate capital and aggregate bequests are calculated, given the optimal policies, and the new values for K and B are compared to the starting values. If they differ by more than a tolerance amount defined in advance, the starting guess is updated, and the algorithm is repeated. The process repeats until the aggregate capital and aggregate bequests reach a steady state.

The solution to the problem is a set of decision rules, $g(a)$ which determine the choices a'. This solution is obtained using backward induction.

5 Numerical Results

5.0.1 Individual Welfare under Social Security

This paper looks into the effects of Social Security on individual welfare, to better understand how, in the face of this subsidy, Social Security remains a highly popular social program across a broad range of income levels. Our first experiments assume a flat 40% replacement rate in the social security benefit formula. This is the same assumption used in Boronow (2007). One key factor in that paper was that low income workers subsidize the old age pension of high income workers (excluding disability and survivor's benefits) under Social Security.

To give a perspective on the size of the subsidy, the flat benefit model aggregate tax rate is 8.77%. The tax rate for high income workers would be 10.04% and for low income workers it would be 7.81%, if it were possible to charge separate fair tax rates. Is it possible that for high income workers the subsidy and the utility of Social Security is enough to gain their support for the program, and for low income workers, does the utility of the program offset the cost of the subsidy enough to garner the support of that type of worker?

We measure individual welfare by the compensating variation in consumption that a newborn in the SSE model economy would need to be indifferent between an economy with or without social security. The approach is to find the percentage increase in annual consumption that equates the value function of a newborn in SSE to the value function of a newborn in the NSSE economy. The results are shown in Table 3.

Table 3. Compensating Variation in SSE-40

	CIU	CBIU
High Income Worker	10.07%	9.15%
Low Income Worker	12.87%	11.83%

When one takes into account the reverse subsidy between high income and low income workers, where the difference in fair value tax rates is 2.23%, then the difference in compensating variations as shown in Table 3 are largely explained. A full explanation of the difference in compensating variations between high and low income workers would also take into account the degree of concavity in the utility function, and the fact that the fair value tax is based on income, while the compensating variation is based on annual consumption. But clearly the reverse subsidy plays a major role in accounting for the difference.

The level of the compensating variations is due to the impact of Social Security on capital formation, which directly affects the wages of both high and low income workers, and indirectly affects welfare through the level of interest rates. Since high income workers benefit disproportionately more than low income workers from the higher interest rates that exist in the low capital intensity SSE economy, this factor, too plays a role in the difference in compensating variations between high and low income workers.

When bequests are in the utillty, the difference in compensating variation between high and low income workers is lessened. One might have expected such a decrease in the compensating variation gap. We know that in the CIU life cycle model, Social Security induces low income workers to reduce savings more than high income workers, which would reduce bequests from low income workers more than hgh income workers. An a priori expectation therefore would be that bequests in utility would have a larger behavioral effect on low income workers. And indeed there is a small decrease in the compensating variation gap between high and low income workers. The relative difference in behavioral response can be noticed in that under CIU, bequests from high income workers were 45.1% of the total bequests, while under the CBIU model, the share of bequests from high income workers drops to 44.1%. (Details are in Table 15A of the Appendix.)

We also notice that with CBIU the level of compensating variation decreases for both high and low income workers. CBIU induces lesser consumption and greater savings behavior, which raises captial formation in the steady state, leading to higher steady state levels of capital, output, wages and consumption, and leads to lower levels of compensating variations.

5.0.2 Diminishing Marginal Utility of Pension Benefits

Another measure of individual welfare differences is willingness-to-pay for the benefits of social security. In this analysis, we measure the marginal willingness-to-pay of high and low income workers when faced with an offer for an additional pension benefit of 1% of lifetime average wage. For the NSSE model, prior to the offer there is no pension, so the 1% addition is the first pension benefit dollar. For the SSE-40 model with a flat 40% replacement rate, the extra 1% brings the pension total to 41%. For the SSE model with a kinked benefit formula, the extra 1% brings the high income replacement rate to 38% and the low income replacement rate to 48%.

For each type of worker, willingness-to-pay is defined as the increase in tax on wages which, combined with the increased pension, would result in zero net change in the value function of a newborn in that economy. Table 4 presents the results when utility is based only on consumption (CIU), and when utility is based on consumption and bequests (CBIU).

Table 4. Willingness-to-pay for Marginal Increase in Public Pension

	NSSE	SSE-40	SSE
Tax Increase WTP	CIU	CIU	CIU
High Income	.214%	.087%	.084%
Low Income	.200%	.083%	.078%
Ratio to Fair Value			
High Income	85.3%	34.7%	33.4%
Low Income	102.5%	42.3%	39.9%
Tax Increase WTP	CBIU	CBIU	CBIU
High Income	.219%	.092%	.090%
Low Income	.198%	.084%	.079%
Ratio to Fair Value			
High Income	87.4%	36.8%	35.8%
Low Income	101.5%	43.1%	40.5%

There are a number of observations to be made from this table. First, the low income worker is willing to pay almost as much as the high income worker is willing to pay (.200% vs. .214%, respectively). This, despite the fact that due to mortality differences, the fair value of the low income worker's benefits is only 77.8% that of the high income worker's benefits. Thus the willingness-to-pay for the added pension benefit is 85.3% of fair value and 102.5% of fair value for high and low incomes types respectively. As expected, low income workers are more willing to pay than high income workers, even to the point of being willing to pay more than the fair value of the extra benefits. But high income workers are not willing to pay the fair value.

Another observation is the significant drop in willingness-to-pay (WTP), between the first dollar of pension benefits (as seen in the NSSE model), and the $41st$ percentage level benefit in SSE-40. As noted above, for the first 1% average wage pension benefit, the WTP is 85.3%/102.5% of fair value (high/low respectively). Yet for the 1% average wage on top of 40% average wage pension benefit, WTP drops to 34.7%/42.3% of fair value. There is clearly a drop in marginal utility going on here. It strongly implies that policy makers looking to modify the present Social Security system should not look to increasing replacement rates. It may also help to explain the relatively mild public response to benefit cuts that were made to Social Security in 1983 and 1996 when benefits that had not been subject to income tax were included in taxable income.

In the SSE model, where the benefit formula is kinked to favor low income workers, there is still more evidence of the decreasing marginal utility of pension benefits. For high income workers, there is a small decrease in WTP from the SSE-40 model. We might have expected a significant drop in WTP, since the high income benefit replacement rate is cut to an average 37%, while the tax rate increases from 8.77% to 9.32%. Yet with lower benefits and higher taxes, the high income WTP only drops from 34.7% of fair value to 33.4% of fair value, a drop of less than 4%. The higher marginal utility experienced at the lower benefit level offsets to some degree the 6.2% higher tax rate and 8% lower benefits. For low income workers, the SSE model with kinked benefit formula results in a higher replacement rate (47%), but with higher taxes at 9.32%. At the higher level of benefits and taxes, marginal utility is lower, resulting in a decrease in WTP from 42.2% to 39.9%, and drop of 5.5%.

A final observation from Table 4 is that when bequests are also in utility, there is an ambiguous effect on WTP. For high income workers, there is a noticeable increase in WTP. For low income workers, there is a decreased WTP for the first dollar of public pension (the NSSE model), but an increased WTP for additional pension benefits in the SSE models. With CBIU, there is a higher steady state level of capital, than with just CIU. (Details are in Table 15A in the Appendix.) Both types of workers benefit from higher wages that result. In the models with Social Security, capital intensity is low, and high income workers can maintain consumption and bequests from high returns on savings. But low income workers cannot benefit as much, not being able to afford as much savings, and must achieve their bequest objective by reducing consumption. Hence in the NSSE case, they have a higher WTP, but in the SSE and SSE-40 cases they have a lower WTP, relative to the CIU experiment.

5.0.3 Progressive (Kinked) Pension Benefit Formula

Diminishing marginal utility of pension benefits makes it unlikely, for plausible values of the preference parameters, that workers would be willing to pay more than the fair value for pension benefits at the margin. But perhaps overall welfare, as measured by the compensating variation, would support the hypothesis, under different parameters. To explore this possibility, we compare compensating variations under SSE with a kinked benefit formula to compensating variations under SSE-40.

Table 5. Compensating Variation Comparisons

	SSE-40 CIU	SSE CIU	Difference
High Income	10.07%	10.99%	.92%
Low Income	12.87%	13.69%	.82%
	CBIU	CBIU	
High Income	9.15%	10.06%	.91%
Low Income	11.83%	12.47%	.64%

We notice that under the kinked benefit formula, the steady state produces an outcome with a larger compensating variation for both types of worker than under the 40% replacement rate. Since replacement rates for low income workers are higher than in the SSE-40 model, low income workers reduce precautionary saving even more. This leads to lower steady state levels of capital, wages, and consumption for the SSE model than for the SSE-40 model (Table 15A in the Appendix). High income workers pay a greater tax and receive less benefits under SSE than under SSE-40, so it is not surprising that their compensating variation would be higher. But low income workers, under the kinked benefit formula receive a bigger slice of this somewhat smaller pie, ending up with a slightly greater level of average annual consumption than in the SSE-40 model. Yet still the compensating variation is greater than in the SSE-40 case, indicating that there is a utility loss from the tax/benefit regime beyond the effect on steady state capital.

5.0.4 Increase in Age at which Benefits Begin

So the kinked benefit formula, while improving equity between the two types of workers, does not move us closer to the hypothesis that workers get enough utility from social security that they would pay more than fair value to live in an economy with social security. So, consider next the effect of increasing the age at which pension benefits begin. Here we take advantage of the known result (Abel, 1985) that it is the insurance mechanism (in the form of pooling survivorship gains) which generates individual welfare gains. We expect then that if pension benefits don't start until an older age, then the insurance mechanism will be more intensive, with a more noticable effect on welfare and WTP. Table 6 presents the compensating variations for the SSE-40 model at alternative benefit starting ages. In addition to the compensating variations, Table 6 also shows the aggregate tax rate needed to provide the pension at the given starting age, as well as the gap between the fair value tax for high and low income workers. Fair value tax rates are the hypothetical tax rate for high income workers to provide their benefits, and the hypothetical tax rate on low income workers to provide their pension benefits. In other words, no subsidy is present. The fair value gap is then a measure of the subsidy embedded in the aggregate tax rate.

Table 6. Compensating Variations by Pension Benefit Starting Age (SSE-40)

Pension Starting Age	65	70	75	80
Utility	CIU	CIU	CIU	CIU
High Income	10.07%	7.64%	5.51%	3.68%
Low Income	12.87%	10.18%	7.64%	5.34%
Difference	2.80%	2.54%	2.13%	1.66%
Aggregate Tax Rate	8.77%	6.01%	3.78%	2.11%
Fair value gap	2.23%	1.95%	1.57%	1.13%
Utility	CBIU	CBIU	CBIU	CBIU
High Income	9.15%	6.78%	4.73%	2.98%
Low Income	11.83%	9.19%	6.73%	4.50%
Difference	2.68%	2.41%	2.00%	1.52%
Aggregate Tax Rate	8.77%	6.01%	3.78%	2.11%
Fair value gap	2.23%	1.95%	1.57%	1.13%

We notice that as the benefit starting age increases, the compensating variations get smaller for both worker types. This is not surprising, since as the age at which pension benefits begin increases, the model economy asymptotically approaches the NSSE economy. We note also that the difference in compensating variation between high and low income workers decreases also, which can in part be attributed to the lessening reverse subsidy embedded in the aggregate tax rate. For the CIU model, we see the compensating variation difference fall from 2.80% at age 65 to 1.66% at age 80. The corresponding reverse subsidy decreases from 2.23% at age 65 to 1.13% at age 80. The remaining difference in compensating variation can be explained in part that high income workers can take better advantage of the higher interest rates that result in a SSE-40 economy, relative to a NSSE economy, and in part that high income workers value the longevity insurance more highly, given their higher "risk" of living to an advanced age.

If we take differences between compensating variation with CIU and compensating variation with CBIU, we obtain a measure of the compensating variation of bequests in utility. For high income workers, bequests in utility reduce compensating variation by .92% at benefit age 65 and by .7% at benefit age 80. For low income workers, the reductions are 1.04% and .84%, respectively. The difference in level of bequests and the concavity of the utility function accounts for the difference between high and low income workers.

Additional insight can be obtained by evaluating the marginal willngness-to-pay (WTP) for pension benefits. Table 7 presents WTP for alternative pension benefit starting ages in a NSSE economy.

Table 7. WTP by Pension Benefit Starting Age (NSSE)

Pension Starting Age	65	70	75	80
Utility	CIU	CIU	CIU	CIU
High Income	.2140%	.1700%	.1324%	.1006%
Low Income	.2000%	.1565%	.1196%	.0883%
Ratio to Fair Value				
High Income	85.3%	95.6%	113.4%	146.2%
Low Income	102.5%	121.2%	154.5%	218.0%
Ratio to Aggregate Tax				
High Income	97.6%	113.3%	140.3%	190.5%
Low Income	91.2%	104.3%	126.7%	167.2%
Utility	CBIU	CBIU	CBIU	CBIU
High Income	.2193%	.1730%	.1329%	.0984%
Low Income	.1981%	.1531%	.1143%	.0810%
Ratio to Fair Value				
High Income	87.4%	97.3%	113.8%	143.0%
Low Income	101.5%	118.6%	147.6%	200.0%
Ratio to Aggregate Tax				
High Income	100.0%	115.2%	140.8%	186.4%
Low Income	90.4%	102.0%	121.1%	153.4%

WTP decreases with starting age, consistent with the decreasing value of a pension that has a later starting age. When related to the fair value however, we see that both types of workers are more willing to pay as the starting age increases. In other words, the utility of the insurance increases faster than the risk nuetral value of the expected payments. The WTP relative to fair value for high income workers (CIU) increases from 85.3% to 146.2% over the range of ages, an increase of 171%. For low income workers, the corresponding increase is 213%, reflecting the greater utility of insurance to low income workers, due to the concavity of the utility function.

WTP relative to the aggregate tax rate, also increases for both types of workers. For high income workers (CIU), the WTP relative to the tax rate increases from 97.6% to 190.5% over the range of ages, an increase of 195%. The corresponding increase for low income workers is 170%. This is less than the 213% increase relative to fair value, because the effect of the reverse subsidy reduces the rate of increase in utility.

WTP that is almost 200% of aggregate tax rates in NSSE model give rise to curiosity as to the result in the SSE-40 model. Will the ratio of WTP to aggregate tax still be greater than 100%, suggesting a path for future reforms? Table 8 presents the results.

Table 8. WTP by Pension Benefit Starting Age (SSE-40)

Pension Starting Age	65	70	75	80
Utility	CIU	CIU	CIU	CIU
High Income	.0869%	.0728%	.0610%	.0495%
Low Income	.0825%	.0678%	.0545%	.0414%
Ratio to Fair Value				
High Income	34.7%	41.0%	52.3%	71.9%
Low Income	42.3%	52.6%	70.4%	102.1%
Ratio to Aggregate Tax				
High Income	39.6%	48.5%	64.6%	93.8%
Low Income	37.6%	45.1%	57.7%	78.4%
Utility	CBIU	CBIU	CBIU	CBIU
High Income	.0921%	.0768%	.0630%	.0489%
Low Income	.0840%	.0677%	.0525%	.0375%
Ratio to Fair Value				
High Income	36.8%	43.2%	53.9%	71.0%
Low Income	43.1%	52.5%	67.8%	92.4%
Ratio to Aggregate Tax				
High Income	42.0%	51.1%	66.7%	92.6%
Low Income	38.3%	45.1%	55.6%	71.0%

WTP drops steadily from the first 1% of average lifetime wage pension to the $41st$% of average lifetime wage pension. At starting age 80, where WTP is greatest, a high income worker (CIU) has a WTP of 146.2% of fair value in Table 7, but only a WTP of 71.9% of fair value in Table 8. Low income workers

(CIU) have a WTP of 102.1% of fair value at age 80, the only instance in Table 8 where the ratio exceeds 100%. In no case does WTP exceed the aggregate tax rate. So again we conclude that diminsihing marginal utility means that workers would not support an increase in benefits and taxes, even at advanced starting ages.

Comparing the CBIU model to the CIU model, we see that at younger starting ages there is a greater WTP, but at older starting ages there is lesser WTP than the corresponding amounts in the CIU models. The difference is more pronounced for low income workers than for high income workers. This is consistent with findings by others that social security reduces bequests, especially for low income workers.

5.0.5 Alternative Risk Aversion Parameters

We turn out attention next to the role of risk aversion. Perhaps a more risk averse worker would be willing to pay more than the fair value of the social security pension. In this model, investment returns are certain, so the role of the risk aversion parameter (γ), is not about investment returns but it is about intertemporal substitution of consumption and savings/bequests. The reciprical of γ is the constant elasticity of intertemporal substitution.

Consider first a partial equilibrium setting, where factor prices are given, taken as the factor prices from the NSSE steady state when $\gamma = 2$. Tables 9A and 9B present the outcomes for agents with alternative elasticities, as they optimize savings and consumption decisions in a partial equilibrium setting.

Table 9A. Alternative Elasticities of Intertemporal Substitution
(NSSE Partial Equilibrium, $K_0 = 6.078, Y = 1.662, r = 2.944\%$)

Elasticity	$\frac{1}{\gamma} = \frac{2}{3}$	$\frac{1}{\gamma} = \frac{1}{2}$	$\frac{1}{\gamma} = \frac{1}{3}$
Savings (K')	5.778	6.078	6.699
Consumption	1.175	1.170	1.162

We notice that as $\frac{1}{\gamma}$ increases, savings decreases. Consumption increases as less is being saved. This reminds us of the situation in the two period model, where, when $R < \frac{1}{\beta\psi}$, the sign of the derivative of c_1 with respect to $\frac{1}{\gamma}$ was positive. Thus consumption moved in the same direction as $\frac{1}{\gamma}$ and savings moved in the opposite direction. That is precisely what we see in Table 9A. In this partial equilibrium, $R\beta = 1.006787$, so ψ has only to be less than .993 for the condition to be satisfied. For high ability agents, the probability of survival to the next period (ψ) is less than .993 for ages after age 57. For low ability agents, the condition is satisfied for ages after 51. In either case, the condition is satisfied well before retirement, at ages where the agent is making significant savings decisions. Thus we might expect savings to decrease, based on intuition from the two period model.

Consider next the situation where the partial equilibrium is based on factor prices from the SSE steady state, when $\gamma = 2$.

Table 9B. Alternative Elasticities of Intertemporal Substitution
(SSE Partial Equilibrium, $K_0 = 4.406, Y = 1.480, r = 5.194\%$)

Elasticity	$\frac{1}{\gamma} = \frac{2}{3}$	$\frac{1}{\gamma} = \frac{1}{2}$	$\frac{1}{\gamma} = \frac{1}{3}$
Savings (K')	5.279	4.406	3.803
Consumption	1.144	1.124	1.108

Here we see the opposite effect from what we observed in the NSSE steady state. As elasticity increases, savings also increases. This is reminiscent of the case in the two period model where $R > \frac{1}{\beta\psi}$. Recall that when $R > \frac{1}{\beta\psi}$, then $\frac{\partial c_1}{\partial \frac{1}{\gamma}} < 0$. In the two period model as $\frac{1}{\gamma}$ increases, c_1 decreases, which causes savings to increase. In Table 9B we also see savings increase as $\frac{1}{\gamma}$ increases. (Consumption also increases, which is unexpected. See below for a discussion on this result.[3]) In the SSE steady state, $R\beta = 1.0288$. Thus to satisfy the condition $R > \frac{1}{\beta\psi}$, ψ must be greater than .972. For high ability workers, this is the situation for ages up to age 72. For low ability workers, $R > \frac{1}{\beta\psi}$ up to age 67. Thus during the working years, when agents are making significant savings decisions, the condition is satisfied that would lead to increased savings for agents with higher elasticities of intertemptoral substitution.

It does not necessarily follow that this same pattern holds in a general equilibrium setting, but it does give intuition as to what to look for in a general equilibrium outcome. In Table 10 we turn our attention to a general equilibrium analysis, allowing factor prices to be determined by the steady state levels of capital. We extend the analysis to also consider the case $\gamma = 4$.

Table 10A. Key Indicators with Alternative Risk Aversion Parameters
(NSSE,CBIU)

Risk Aversion Parameter	$\frac{1}{\gamma} = \frac{2}{3}$	$\frac{1}{\gamma} = \frac{1}{2}$	$\frac{1}{\gamma} = \frac{1}{3}$	$\frac{1}{\gamma} = \frac{1}{4}$
Capital	5.923	6.078	6.724	7.648
Consumption	1.167	1.170	1.179	1.186
Wage	1.315	1.327	1.377	1.442
Interest Rate	3.11%	2.94%	2.33%	1.60%

Table 10B. Key Indicators with Alternative Risk Aversion Parameters
(SSE,CBIU)

Risk Aversion Parameter	$\frac{1}{\gamma} = \frac{2}{3}$	$\frac{1}{\gamma} = \frac{1}{2}$	$\frac{1}{\gamma} = \frac{1}{3}$	$\frac{1}{\gamma} = \frac{1}{4}$
Capital	4.698	4.407	4.033	3.800
Consumption	1.135	1.124	1.108	1.096
Wage	1.210	1.182	1.145	1.121
Interest Rate	4.71%	5.19%	5.90%	6.40%

[3] As noted above, average annual consumption increased along with increased saving and higher elasticity in the SSE setting. One would expect consumption to decrease if savings increase. However, in the SSE steady state, interest rates are high enough that increased savings earns interest that funds enough consumption to offset the consumption lost to higher savings. Likewise, as $\frac{1}{\gamma}$ falls, savings decreases and consumption increases. But the interest not earned on savings that does not occur is enough to offset the extra consumption that otherwise would have gone to savings. Overall there is a decrease in consumption.

In the NSSE model of Table 10A, increasing γ decreases the elasticity of intertemptoral substitution and causes an increase in saving, wages and consumption. But in Table 10B, under the SSE model, a decrease in the elasticity of intertemporal substitution causes a decrease in saving, wages and consumption. This result was also obtained by Imrohoroglu, Imrohoroglu and Joines (Time Inconsistent Preferences). They commented, "a low elasticity of substitution also raises the costs of social security, which takes the form of lower steady-state capital and lifetime consumption". They also commented, "The smaller the elasticity of substitution, the greater the reduction in the saving of workers when government attempts to reallocate consumption toward retirement years through the payroll tax." Based on the intuition of the two period and partial equilibrium models, we trace the source of the response to the relationship between the return on investment, the probabilility of survival and the time preference parameter.

To gain further insight to this finding, we turn first to a measure of overall welfare, the compensating variation in compensation. Table 11 presents the compensating variation for agents born into the SSE under alternative values for the elasticity of intertemptoral substitution.

Table 11. Compensating Variations and Elasticity of Intertemptoral Substitution

Risk Aversion Parameter	$\frac{1}{\gamma}=\frac{2}{3}$	$\frac{1}{\gamma}=\frac{1}{2}$	$\frac{1}{\gamma}=\frac{1}{3}$	$\frac{1}{\gamma}=\frac{1}{4}$
Utility	CIU	CIU	CIU	CIU
High Income	7.5%	11.0%	18.1%	27.4%
Low Income	10.9%	13.7%	21.1%	31.0%
Utility	CBIU	CBIU	CBIU	CBIU
High Income	6.5%	10.1%	18.1%	32.9%
Low Income	9.3%	12.5%	22.0%	39.4%

The contrast in saving behavior as the elasticity of substitution decreases between the NSSE economy and the SSE economy is evident in Table 11. We saw in Tables 10A and 10B that in the NSSE economy, consumption and capital increase with a decrease in $\frac{1}{\gamma}$, while just the opposite occurs in the SSE economy. Thus we would expect that the compensating variation would increase as γ increases (and $\frac{1}{\gamma}$ decreases). As Table 11 shows, the compensating variation increases quite dramatically. Moreover the difference between high income and low income worker's compensating variation widens. Under the SSE economy, at higher levels of γ, there is a scarcity of capital and high interest rates which favors high income workers relative to low income workers. This accounts for the widening gap. The data from the CBIU experiment relative to the CIU experiment, shows the widening gap even more clearly, confirmimg that high income workers are better able to sustain a bequest, thanks to higher interest rates.

As before, we now look to the marginal WTP for further insight into individual welfare. Table 12 presents the WTP results for alternative elasticities of substitution, when utility includes bequests.

Table 12A. WTP and Elasticity of Intertemptoral Substitution
(NSSE, CBIU)

Risk Aversion Parameter	$\frac{1}{\gamma}=\frac{2}{3}$	$\frac{1}{\gamma}=\frac{1}{2}$	$\frac{1}{\gamma}=\frac{1}{3}$	$\frac{1}{\gamma}=\frac{1}{4}$
High Income	.2071%	.2193%	.2455%	.2629%
Low Income	.1843%	.1981%	.2230%	.2360%
Ratio WTP to Fair Value				
High Income	82.5%	87.4%	97.8%	104.8%
Low Income	94.4%	101.5%	114.2%	120.9%
Ratio WTP to Aggregate Tax				
High Income	94.5%	100.0%	112.0%	119.9%
Low Income	84.1%	90.4%	101.7%	107.7%

Table 12B. WTP and Risk Aversion Parameters
(SSE, CBIU)

Risk Aversion Parameter	$\frac{1}{\gamma}=\frac{2}{3}$	$\frac{1}{\gamma}=\frac{1}{2}$	$\frac{1}{\gamma}=\frac{1}{3}$	$\frac{1}{\gamma}=\frac{1}{4}$
High Income	.1075%	.0897%	.0535%	.0252%
Low Income	.1064%	.0791%	.0479%	.0222%
Ratio WTP to Fair Value				
High Income	42.8%	35.8%	21.3%	10.0%
Low Income	54.5%	40.5%	24.5%	11.4%
Ratio WTP to Aggregate Tax				
High Income	49.0%	40.9%	24.4%	11.5%
Low Income	48.5%	36.1%	21.9%	10.1%

There are two quite different pictures emerging from Tables 12A and 12B. In Table 12A, WTP increases as $\frac{1}{\gamma}$ decreases. The low returns on investment in the NSSE make the returns to survivorship relatively more attractive and raises WTP. For $\gamma = 4$, both types of workers are willing to pay more than the fair value to obtain a pension benefit of 1% of average lifetime wage. Looking at WTP as a percentage of the aggregate tax, we see that for $\frac{1}{\gamma} = \frac{1}{3}$, and for $\frac{1}{\gamma} = \frac{1}{4}$, both types of workers are willing to pay more than the aggregate tax, taking into account the subsidy to high income workers! However, when we look to the SSE model under various levels of γ, we find that WTP decreases as $\frac{1}{\gamma}$ decreases. Here the returns on investment are more attractive than the returns on survivorship, decreasing the WTP. As a result, WTP as a percentage of the aggregate tax never exceeds 50%.

5.0.6 Welfare Under Temptation

Research by Imrohoroğlu, Imrohoroğlu, and Joines (2003), Kumru and Thanopoulos (2007), and Boronow (2007) all present evidence that Social Security is a weak commitment device. But is even a weak commitment device enough to raise welfare so that the status quo is rational? To evaluate that hypothesis, we return to the model of Temptation in the earlier paper by Boronow (2007), to compare welfare outcomes for the various model economies.

In that earlier paper, Boronow evaluated outcomes in a NSSE model, a SSE-40 model and a proposed Hybrid Reform Economy (HRE) model. The HRE model has the same pension replacement rate as the SSE-40 model, but benefits begin at age 80 instead of age 65. There is a mandatory personal security account (PSA) commitment device, which is accumulated during working years from a payroll contribution, and amortized ratably during the period from retirement to the start of the public pension at age 80. The contribution rate is chosen so that the contribution rate plus the tax to provide the advanced age public pension sum to the same total as the tax rate in the SSE-40 model. In this paper, I also present results for a model denoted by SSE-40 @ age 80. This is the same as the HRE model, but without the device of the PSA. It is also the same as the SSE-40 model, but with the pension starting at age 80 instead of age 65 (and therefore with lower labor tax). This additional model will help reveal the effect of the PSA commitment device.

Under standard preferences in which agents know and act according to their best interests, there is no need for a commitment device. However, if agents are subject to temptation preferences, they make choices which are not in their best long run interests. We use the model of choices under temptation used by Gul and Pesendorfer (2001) and Krusell, Kuruscu, and Smith (2005) and Boronow (2007). For a complete description of the temptation model, see Boronow (2007). A brief description is in the Appendix. As in the 2007 paper, results are presented for three different specifications of temptation (labeled weak, moderate and strong), corresponding to increasing the temptation stength parameter.

The model is only computed for the case where utility is from consumption (CIU). [4]

Table 13. Compensating Variation under Temptation (CIU)

[4] The theoretical algorithm to compute results for the temptation model with CBIU is available from the author, but results were not produced, as the programming and computational costs did not seem worthwhile in view of the insignificant difference between CIU outcomes and CBIU outcomes so far.

	High Income	Low Income	Difference
SSE-40 @ Age 65			
Standard Preferences	9.59%	12.77%	3.18%
Weak Temptation	10.71	12.72	2.01
Moderate Temptation	10.14	11.88	1.74
Strong Temptation	9.61	11.28	1.67
SSE-40 @ Age 80			
Standard Preferences	3.41%	5.23%	1.82%
Weak Temptation	4.22	5.06	.84
Moderate Temptation	3.49	4.34	.85
Strong Temptation	3.18	4.05	.87
HRE			
Standard Preferences	4.07%	5.84%	1.77%
Weak Temptation	1.94	1.08	−0.86
Moderate Temptation	−4.66	−6.31	−1.65
Strong Temptation	−10.21	−12.06	−1.85

In the SSE-40 case, we see that low income workers are more adversely affected by social security, in that they require a higher compensating variation in consumption to equate their welfare to that in the NSSE economy. The subsidy from low income to high income workers, and the higher interest rates in the SSE-40 economy relative to the NSSE economy both benefit high income workers, not low income workers. Hence the outcome is not surprising. But we notice that the difference in CV between low income workers and high income workers gets smaller as the temptation setup gets stronger. The difference is 3.18% under standard preferences but only 1.67% under the strongest specification of temptation. This indicates that the changes to welfare under temptation for high income workers are relatively greater than for low income workers. Perhaps this is due to the higher interest rates under temptation, which affect high income workers disproportionaltely. (Key indicators for the steady state outcomes under temptation are shown in the Appendix.) Under the SSE-40@85 model, the difference in CV between high income and low income workers is flat, or even slightly increasing in strength of temptation. This also suggests that high interest rates are the main driver of the change in the difference, since in the SSE-40@85 model, both types of workers must save for the early retirment years, so high interest rates benefit both about the same.

Another notable feature of SSE-40 and SSE-40@85 models in Table 13 is that compensating variations decrease as the strength of temptation increases. This is evidence of the commitment value in social security. As temptation is stronger, the agent values the freedom of the NSSE economy less highly. This is true for both high and low income workers.

There is a different situation for the HRE model, however. For both types of agents, their CV is positive under standard preferences, indicating the agents prefer the NSSE model to the HRE model. But under temptation, the CV is smaller (weak specification) and eventually is negative (moderate and strong specifications), indicating that agents prefer the HRE model to the NSSE model. This is evidence that the PSA is an effective commitment device, preserving savings, capital, wages and welfare under temptation. (Recall that the only difference between the HRE model and the SSE-40@85 model is that the HRE mandates the accumulation and depletion of a PSA device, whereas the SSE-40@85 leaves the saving/consumption decision to individual utility optimization.)

Not only is the HRE model preferred to the NSSE model, but the usual pattern where the CV of low income workers is greater than high income workers is broken under temptation. High income workers, having more voluntary savings, are more adversely affected by temptation than low income workers. With the HRE model, low income workers benefit from high interest returns on their mandatory PSA, while high income workers reduce voluntary savings due to temptation. CVs under temptation indicate low income workers are relatively happier with HRE than are high income workers (who also prefer HRE to NSSE, just not as much).

Increased overall welfare under HRE, and the evidence of Social Security as a commitment device, raise the possibility that willingness to pay may also be higher, maybe even high enough to pay the required tax. Table 14 presents results about willingness to pay as a percentage of the actuarially required tax.

Table 14. WTP and Temptation Preferences (CIU)
(as % of actuarially required tax)

	NSSE	SSE-40@65	SSE-40@80	HRE
High Income Worker				
Standard Preferences	105.3%	39.1%	93.8%	98.9%
Weak Temptation	87.1	36.5	88.8	113.5
Moderate Temptation	76.3	33.7	78.9	118.9
Strong Temptation	68.1	31.7	72.6	130.2
Low Income Worker				
Standard Preferences	87.0%	37.9	78.4%	78.3
Weak Temptation	79.2	35.7	73.5	69.5
Moderate Temptation	69.0	33.0	68.3	70.9
Strong Temptation	60.4	30.9	62.9	76.4

In the NSSE model, the usual result is obtained, that there is a high willingness to pay for the first marginal dollar of pension. But when temptation is introduced in the setup, WTP decreases as temptation strength is increased. If the commitment device embedded in Social Security had a high utility value, we would expect to see WTP increase with the strength parameter of temptation. So the commitment device of a paygo pension under temptation is not the explanation of the status quo problem. In fact, temptaton makes the popularity of social security even harder to explain. This is evident in the results for the SSE model, where the combination of diminishing marginal utility of pension benefits and the deteriorating effect of temptation reduce WTP to about 31% of the actuarially required tax , from about 38% without temptation. Starting the social security benenfits at an advanced age increases WTP, but the pattern is still the same, as seen in the SSE-40@85 model results.

But under HRE, the pattern is different. Without temptation, WTP is about the same as the SSE-40@80 model. WTP in the HRE model (98.9% for high income workers) is higher than WTP in the SSE-40 model (39.1% for high income workers). This is due to the advanced starting age, which raises the insurance value of the pension, as we saw in an earlier section. But when temptation is included in the setup, we find that WTP increases as temptation strength increases. High income workers, who are subsidized by low income workers, are willing to pay more than the aggregate tax rate. Low income workers are willing to pay more than their fair share, but less than the aggregate tax rate which includes the subsidy. Even so, they are willing to pay 70% or more of the tax rate, getting close to the situation where both types of agents are WTP the required tax. It is not unreasonable to think that with a kinked benefit formula to reduce the subsidy, that both types of agents would be willing to pay the required tax.

6 Conclusions

When income is correlated with longevity, then low income workers will have a shorter expected lifetime than high income workers. Yet they pay the same tax rate as high income workers under Social Security. Thus there is an inherent disadvantage for low income workers. We also know that concave utility functions assign greater utility from insurance to low income workers than to high income workers. Assuming utility is concave, could these effects offset each other enough to offer an explanation as to why Social Security remains a popular program at all income levels? We examined this hypothesis under alternative ages at which pensions begin (to increase the intensity of the insurance element), under a progressive benefit formula (to reduce the subsidy), and under alternative parameters for risk aversion (which changes the concavity of the utility function). Finally, we examined the hypothesis under a setup in which agents are subject to temptation preferences. Although we obtained some tantalizing hints of happy voters in the NSSE model, in every case the effects of diminishing marginal utility of pension benefits reduced overall individual welfare to levels well below what would explain continued support for Social Security. It would seem therefore that the status quo problem is truly that Social Security is simply less bad than the alternatives (so far).

We did find that the Hybrid Reform proposal produced increasing welfare under temptation, as measured by the compensating variation in consumption. For temptation that is sufficiently strong, agents prefer the Hybrid Reform social insurance mechanism to the economy with no social security. Also, the Hybrid Reform produced the highest ratio of WTP to required tax. Under temptation, high income agents were willing to pay more than the required tax, and low income workers were willing to pay most of the required tax. It seems very likely that with a kinked benefit design to reduce the subsidy, it would be possible for both types of workers to be WTP more than the required tax rate, if temptation is indeed a valid framework. So, while we were not able to find a welfare explanation for the popularity of social security, we did produce evidence that if agents are indeed subect to temptation preferences, that the HRE device would serve well in preserving welfare.

7 Appendix

7.0.7 Appendix A: Steady State Values for Key Indicators

Table 15A. Key Indicators ($\gamma = 2$)

		NSSE	SSE-40	SSE	NSSE	SSE-40	SSE
Preferences		CIU	CIU	CIU	CBIU	CBIU	CBIU
Capital	K	5.757	4.205	4.142	6.078	4.465	4.407
Output	Y	1.630	1.456	1.448	1.662	1.487	1.480
Consumption	C	1.164	1.115	1.113	1.170	1.126	1.124
Cons: High	$C_{(1)}$	.503	.494	.502	.504	.496	.493
Cons: Low	$C_{(2)}$	.661	.622	.646	.666	.630	.631
Wage	w	1.302	1.162	1.156	1.327	1.188	1.182
Interest Rate	r	3.29%	5.56%	5.68%	2.94%	5.09%	5.19%
Bequest	Beq	.1116	.0762	.0742	.1164	.0854	.0838
Beq: High	$Beq_{(1)}$	.0488	.0343	.0363	.0503	.0377	.0392
Beq: Low	$Beq_{(2)}$	.0628	.0419	.0379	.0661	.0477	.0446

Table 15B. Key Indicators at Alternative Benefit Ages ($\gamma = 2$, SSE-40,CIU)

		Age 65	Age 70	Age 75	Age 80
Preferences		CIU	CIU	CIU	CIU
Capital	K	4.205	4.422	4.660	4.914
Output	Y	1.456	1.482	1.510	1.540
Consumption	C	1.115	1.124	1.133	1.142
Cons: High	$C_{(1)}$	.494	.496	.498	.500
Cons: Low	$C_{(2)}$	.622	.628	.635	.642
Wage	w	1.162	1.184	1.206	1.230
Interest Rate	r	5.56%	5.16%	4.77%	4.38%
Bequest	Beq	.0762	.0754	.0769	.0810
Beq: High	$Beq_{(1)}$	.0343	.0335	.0333	.0342
Beq: Low	$Beq_{(2)}$	.0419	.0419	.0436	.0468

Table 15C. Key Indicators at Alternative Benefit Ages ($\gamma = 2$, SSE-40,CBIU)

		Age 65	Age 70	Age 75	Age 80
Preferences		CBIU	CBIU	CBIU	CBIU
Capital	K	4.465	4.711	4.980	5.270
Output	Y	1.487	1.516	1.547	1.579
Consumption	C	1.126	1.135	1.144	1.152
Cons: High	$C_{(1)}$	.496	.498	.500	.502
Cons: Low	$C_{(2)}$	.630	.637	.644	.650
Wage	w	1.188	1.211	1.235	1.261
Interest Rate	r	5.09%	4.69%	4.28%	3.88%
Bequest	Beq	.0854	.0848	.0866	.0911
Beq: High	$Beq_{(1)}$	.0377	.0368	.0368	.0379
Beq: Low	$Beq_{(2)}$	.0477	.0480	.0498	.0532

Table 15D. Key Indicators at Alternative Elasticities of Intertemporal Substituion (NSSE,CIU)

		log util.	$\frac{1}{\gamma}=\frac{2}{3}$	$\frac{1}{\gamma}=\frac{1}{2}$	$\frac{1}{\gamma}=\frac{1}{3}$	$\frac{1}{\gamma}=\frac{1}{4}$
Preferences		CIU	CIU	CIU	CIU	CIU
Capital	K	5.511	5.597	5.757	6.268	7.003
Output	Y	1.604	1.613	1.630	1.681	1.749
Consumption	C	1.159	1.161	1.164	1.173	1.182
Wage	w	1.281	1.288	1.302	1.342	1.397
Interest Rate	r	3.58%	3.48%	3.29%	2.75%	2.09%
Bequest	Beq	.0898	.1020	.1116	.1280	.1435

Table 15E. Key Indicators at Alternative Elasticities of Intertemporal Substituion (NSSE,CBIU)

		$\frac{1}{\gamma}=\frac{2}{3}$	$\frac{1}{\gamma}=\frac{1}{2}$	$\frac{1}{\gamma}=\frac{1}{3}$	$\frac{1}{\gamma}=\frac{1}{4}$
Preferences		CBIU	CBIU	CBIU	CBIU
Capital	K	5.923	6.078	6.724	7.648
Output	Y	1.646	1.662	1.724	1.805
Consumption	C	1.167	1.170	1.179	1.186
Wage	w	1.315	1.327	1.377	1.442
Interest Rate	r	3.11%	2.94%	2.33%	1.60%
Bequest	Beq	.1100	.1164	.1320	.1490

Table 15F. Key Indicators at Alternative Elasticities of Intertemporal Substituion (SSE,CIU)

		$\frac{1}{\gamma}=\frac{2}{3}$	$\frac{1}{\gamma}=\frac{1}{2}$	$\frac{1}{\gamma}=\frac{1}{3}$	$\frac{1}{\gamma}=\frac{1}{4}$
Preferences		CIU	CIU	CIU	CIU
Capital	K	4.387	4.142	3.738	3.410
Output	Y	1.478	1.448	1.395	1.350
Consumption	C	1.123	1.113	1.093	1.074
Wage	w	1.180	1.156	1.114	1.078
Interest Rate	r	5.23%	5.69%	6.537%	7.35%
Bequest	Beq	.0699	.0742	.0787	.0800

Table 15G. Key Indicators at Alternative Elasticities of Intertemporal Substituion (SSE,CBIU)

		$\frac{1}{\gamma}=\frac{2}{3}$	$\frac{1}{\gamma}=\frac{1}{2}$	$\frac{1}{\gamma}=\frac{1}{3}$	$\frac{1}{\gamma}=\frac{1}{4}$
Preferences		CBIU	CBIU	CBIU	CBIU
Capital	K	4.986	4.407	4.033	3.800
Output	Y	1.515	1.480	1.434	1.403
Consumption	C	1.135	1.124	1.108	1.096
Wage	w	1.21	1.182	1.145	1.121
Interest Rate	r	4.71%	5.19%	5.90%	6.40%
Bequest	Beq	.0853	.0838	.0832	.0833

Table 15H. Key Indicators under Temptation (NSSE,CIU)

Preferences		Standard	Weak Tempt.	Mod. Tempt.	Str. Tempt.
Capital	K	5.720	3.830	2.813	2.235
Output	Y	1.626	1.407	1.260	1.159
Consumption	C	1.163	1.097	1.032	.979
Wage	w	1.299	1.124	1.006	.926
Interest Rate	r	3.33%	6.33%	9.22%	11.77%
Bequest	Beq	.1113	.0865	.0702	.0598

Table 15H. Key Indicators under Temptation (SSE-40,CIU)

Preferences		Standard	Weak Tempt.	Mod. Tempt.	Str. Tempt.
Capital	K	4.204	2.980	2.314	1.898
Output	Y	1.455	1.286	1.174	1.093
Consumption	C	1.115	1.044	.987	.940
Wage	w	1.162	1.027	.937	.873
Interest Rate	r	5.56%	8.63%	11.36%	13.83%
Bequest	Beq	.0760	.0614	.0522	.0456

Table 15H. Key Indicators under Temptation (SSE-40@Age85,CIU)

Preferences		Standard	Weak Tempt.	Mod. Tempt.	Str. Tempt.
Capital	K	4.914	3.387	2.589	2.094
Output	Y	1.540	1.347	1.222	1.132
Consumption	C	1.142	1.072	1.023	.963
Wage	w	1.230	1.075	.976	.904
Interest Rate	r	4.38%	7.41	10.10	12.57
Bequest	Beq	.0810	.0647	.0548	.0476

Table 15H. Key Indicators under Temptation (HRE,CIU)

Preferences		Standard	Weak Tempt.	Mod. Tempt.	Str. Tempt.
Capital	K	5.091	4.096	4.010	3.996
Output	Y	1.559	1.442	1.431	1.429
Consumption	C	1.147	1.110	1.106	1.105
Wage	w	1.245	1.152	1.143	1.141
Interest Rate	r	4.13%	5.77%	5.95%	5.97%
Bequest	Beq	.0801	.0711	.0707	.0696

7.0.8 Appendix B: Temptation

Intuition Temptation preferences are developed by Gul and Pesendorfer in a series of articles in 2001, 2002, and 2004. Below is a summary of the intuition of temptation preferences.

The setting consists of an individual representative agent who will choose today a set of alternatives B, among which she will choose a consumption lottery next period. Assuming that the agent is an expected utility maximizer, the next period the agent chooses lottery $p \in B$ which solves $\max_{p\in B} \int u(p)dp$. At time 0, our agent prefers the set of alternatives B to the set of alternatives B' when $\max_{p\in B} \int u(p)dp \geq \max_{p\in B'} \int u(p)dp$. In the theory of expected utility, a standard axiom is that $B \succsim B' \Rightarrow B \sim B \cup B'$ (Kreps (1979)). This axiom rules out situations where the agent may benefit from or be harmed by the addition of alternatives in B'. The idea of temptation preferences is that the agent may strictly prefer B to $B \cup B'$, knowing that B' contains temptations to which she will be vulnerable in time 1. Thus, in the Gul and Pesendorfer development of temptation preferences, the standard axiom is relaxed to the 'set betweenness' axiom:

$$B \succsim B' \Rightarrow B \succsim B \cup B' \succsim B'$$

With the 'set betweenness' axiom, if $B \succsim B \cup B'$ it is possible for the agent at time 0 to strictly prefer the smaller set B of time 1 choices, to the larger set $B \cup B'$ which contains tempting choices that the agent would rather not face at time 1. The idea here is that the presence of temptations at time 1 are the reason for the preference for commitment to a choice set at time 0. To avoid the temptation, at time 0 our agent strictly prefers the set of alternatives B. In fact, this is the criteria by which a temptation is identified.

As Gul and Pesendorfer (2001) show, the set betweenness axiom together with the other axioms that otherwise lead to standard expected utility, lead instead to the utility representation of temptation preferences. The utility representation is as follows. Let $u(p)$ and $v(p)$ be von Neumann-Morgenstern expected utility functions. Intuitively, u is the utility function that is not affected by temptation, and v is the utility function that is affected by temptation. Then temptation preferences are represented by the function:

$$U(B) = \max_{p\in B} \int (u(p) + v(p))dp - \max_{\widetilde{p}'\in B} \int v(\widetilde{p})d\widetilde{p}$$

Note that when the agent has only one choice, then the function u is the agent's utility; the v terms net to zero. Thus Gul and Pesendorfer refer to $u(p)$ as 'the commitment utility', since it represents utility when one is committed to p (i.e., $B = \{p\}$). It is worth noting here that an agent with perfect foresight would also have utility u, and the standard assumption $B \sim B \cup B'$ would apply. An agent with perfect foresight is not vulnerable to temptation; for such a decision-maker $v = u$ and therefore the utility formulation above collapses to the usual expected utility function. Thus other authors refer to u as 'normative' utility, which is the terminology used in this paper.

The utility function v represents the temptation utility. In the model presented by Gul and Pesendorfer (2002), the agent maximizes $u + v$, arriving at the optimal compromise, with the expenditure of self-control with disutility

$$v(c) - \max_{\widetilde{p} \in B} \int v(\widetilde{p}) d\widetilde{p}$$

where c is $\arg\max_{p \in B} \int (u(p) + v(p)) dp$.

As Gul and Pesendorfer (2002) point out, temptation is closely related to the literature on time-inconsistent preferences. Time-inconsistent preferences lead to a solution strategy in which multiple selves are players in a game to find an equilibrium which is consistent. Krusell, Kuruşçu and Smith (2005) show that the Phelps-Pollack form of time-inconsistent preferences are equivalent to temptation preferences under a setup where the strength of the temptation goes to infinity.

An important feature of temptation preferences, and the reason they are useful in this analysis of Social Security, is that unlike the Phelps-Pollack formulation of time inconsistent preferences, temptation preferences are amenable to a dynamic setting. In the finite horizon case, the setting is that an agent chooses a consumption lottery from a set of alternatives B_t, and chooses a decision problem set B_{t+1} for the next period. Formally,

$$W_{t-1}(B_t) = \max_{p \in B_t} \int [u_t(p) + W_t(p, B_{t+1}) + V_t(p, B_{t+1})] dp - \max_{\widetilde{p} \in B_t} \int V_t(\widetilde{p}, B_{t+1}) d\widetilde{p}$$

In this representation, W_{t-1} represents the value function of the agent over the period t choices; prior to period t, the agent is committed to choose from among the choices in B_t. The normative (commitment) utility is $u_t + W_t$, and the temptation utility is V_t. In the final period (period T), the decision problem set is a singleton set, typically $\{0\}$ or the empty set (i.e., $B_{T+1} = \{0\}$).

$$W_{T-1}(B_T) = \max_{p \in B_T} \int [u_T(p) + 0 + V_T(p)] dp - \max_{\widetilde{p} \in B_T} \int V_T(\widetilde{p}) d\widetilde{p}$$

Setup This paper uses the same temptation model as Boronow (2007), which uses the setup of Krusell, Kuruşçu and Smith (2005).

Let U denote normative utility and V denote temptation utility. The normative utility is given by:

$$U_j = \sum_{t=0}^{J-j} \beta^t [\Psi_{j,t}(z)] \cdot u(c_{j+t}(z))$$

Temptation utility is specified in terms of the felicity function u, and two parameters, a strength parameter σ and a future discount parameter φ:

$$V_j(\widetilde{c}, a, \widetilde{a}')) = \sigma[u(\widetilde{c}) + \varphi\beta\psi_j(z)W_{j+1}(\widetilde{a}')]$$

Putting this all together, an agent with temptation preferences maximizes:

$$\begin{aligned} W_j(a) &= \max_{c,a'} \{U_j(c, a, a') + V_j(c, a, a')\} - \max_{\widetilde{c},\widetilde{a}'} V_j(\widetilde{c}, a, \widetilde{a}')) \\ &= \max_{c,a'} \{(1+\sigma)u(c) + (1+\sigma\varphi)\beta\psi_j(z)W_{j+1}(a')\} \\ &\quad - \max_{\widetilde{c},\widetilde{a}'} \sigma\{u(\widetilde{c}) + \varphi\beta\psi_j(z)W_{j+1}(\widetilde{a}')\} \end{aligned}$$

subject to the same conditions as under standard preferences. Note that c and $\widetilde{c}$ are given by:

$$\begin{aligned} c &= (a+\xi)\cdot(1+r) + Q - a' \\ \widetilde{c} &= (a+\xi)\cdot(1+r) + Q - \widetilde{a}' \end{aligned}$$

In the model used in this paper, the temptation parameters (σ, φ) are parameterized as $(.1, .7), (.2, .7)$, and $(.3, .7)$, representing weak, moderate and strong temptation respectively.

8 References

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