A Hybrid Reform: Social Security on Dual Power in the Presence of Temptation*

Gordon C. Boronow
Stony Brook University

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Abstract

Heterogeneous ability/income which is correlated with longevity is incorporated into a large scale general equilibrium overlapping generations economy. We evaluate the effects of a hybrid refom of Social Security which includes (i) a paygo advanced old age government pension that enhances the security of the old retirees, and (ii) personal accounts to finance the early retirement ages. These policy reforms are studied in a life cycle consumer framework where households face temptation and self-control problems. Our analysis indicates that the economy with the hybrid reform is able to sustain higher levels of capital intensity, and therefore consumption, than either an economy with Social Security or an economy with no Social Security. In addition, the proposed hybrid reform reduces wealth inequality.

1 Introduction

One of the most important issues studied in the macroeconomics literature is Social Security reform and its subsequent effect on the macroeconomy.[1] As a result, we have a sound understanding of many issues related to Social Security, and the mechanisms by which Social Security affects the national economy. It is an indication of the inherent complexity of Social Security in the political economy, that even with this broad understanding there is still not a corresponding consensus as to the best way to address the coming financial meltdown in Social Security as the Baby Boomer generation reaches retirement.

There are two basic strategies for reforming Social Security, and many intermediate mixtures of these two. One strategy is for individuals to take responsibility for their own retirement. A pure version on this theme would be

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[1]For this paper, Social Security will only refer to old age pension benefits and exclude disability and survivor benefits.

total elimination of Social Security. Another variation would require each individual to build up a Personal Security Account (PSA) through mandated contributions. In either variation, the distinguishing characteristic is individual responsibility, ownership and freedom of action for their own retirement income. The other basic approach is for the government to continue to have responsibility for providing a retirement pension to qualified workers. Given the current actuarial imbalance, reform proposals under this approach necessarily involve some variation of tax increase, or benefit reduction to "fix" Social Security. But the distinguishing characteristic of this approach is government action with no choice or responsibility on the part of the individual.

This paper postulates a "hybrid" reform, one of many possible intermediate mixtures between the two basic approaches. The Hybrid Reform proposal is motivated by the possibility that the optimal solution to Social Security is not a corner solution, but instead takes design features from each of the two basic approaches, based on what we already have learned.

From the present government-based system, the Hybrid Reform proposal retains a government provided pension, but one which starts at an advanced old age (in particular, at age 80). We know that in a partial equilibrium setting, a government-provided pension can be a welfare-enhancing benefit for individuals (Abel 1985). It reduces retirement income uncertainty, and protects citizens when they are not necessarily able to fend for themselves. But, as discussed below in the related literature, we have also learned that the reduction in income uncertainty in a life cycle model leads to lower precautionary savings, and less capital formation in the economy. In general equilibrium models of the economy, lower rates of savings lead to lower levels of output, wages and individual welfare.

From the individual responsibility approach, The Hybrid Reform proposal incorporates a mandatory Personal Security Account (PSA) to accumulate assets that will provide retirement income in the years before the government pension starts. By eschewing (in part) intergenerational transfers in favor of accumulated assets, the Hybrid Reform results in increased saving (and greater capital formation), which in equilibrium leads to a higher standard of living. An approach that relies on individual responsibility has problems too, such as the free rider problem, whereby some people may not save for retirement in the expectation that society will not let them starve. In this paper, this is managed by making contributions to the PSA mandatory in nature.

The peculiar design of the Hybrid Reform, which has the pension start at a much later age than we are accustomed to, deserves a comment as to its motivation. It is not uncommon in the group insurance businesses, that larger companies self-insure their group health insurance benefits. If they do so, they also purchase additional insurance coverage against the tail of the claim distribution, as a matter of risk management. In this way, they are fairly certain what their costs will be, and they are not exposed to an unlimited risk. Similarly, the early retirement years of individuals are such a candidate for self-insurance. Since life expectancy is higher (Boronow (2006)), we can self-insure the early retirement years with a reasonable degree of certainty. In addition, we can insure for the late retirement years, which have a high degree of uncertainty, with a public pension. By self-insuring, we gain more freedom of choice over the optimal use of our resources, and hope to enhance welfare.

The paper compares first the stationary state in economies under Social Security, No Social Security, and the proposed Hybrid Reform assuming standard preferences. Next, the same analysis is conducted under temptation preferences, as developed by Gul and Pesendorfer (2000). In particular, we use the formulation of Krusell, Kuruşçu and Smith (2005), assuming that agents are tempted by a higher level of current consumption relative to future consumption. The framework for our analysis is a large-scale general equilibrium overlapping generations (OLG) model. This setting, in various permutations has been used extensively since Auerback and Kotlikoff (1987) first analyzed labor supply and capital stock with a 55-period deterministic OLG model.[2] Like Fuster, Imrohoroğlu, Imrohoroğlu (2002), we assume that agents are endowed with heterogeneous ability and correlated mortality.

Our main results can be summarized as follows. First, the paper makes the argument that the Hybrid Reform economy (HRE), using the dual power of a government provided pension and mandatory, self-funded Personal Security Accounts (PSA), does a better job of protecting the macroeconomy and enhancing welfare in the long run than either the Social Security economy (SSE), which has only a government pension, or the No Social Security economy (NSSE), which has only voluntary savings. This takes into account the effects of temptation preferences, which induce agents to struggle with self-control issues. The argument that HRE produces a better outcome in the long run is based on the fact that aggregate welfare is higher for the HRE model under standard preferences (i.e. corresponding to agents with perfect foresight), while this welfare level is sustained under temptation preferences (i.e. corresponding to agents with myopic foresight). In contrast, the SSE model leads to the lowest long run welfare in the three model economies under standard preferences, and long run welfare decreases even more with temptation preferences. Finally, the NSSE model leads to the highest long run welfare with perfect foresight, but welfare falls even more rapidly than in the SSE in the presence of temptation. In sum, only the HRE model sustains the level of savings and individual welfare with temptation preferences.

Second, this paper shows that the Hybrid Reform economy generates less wealth inequality than does a Social Security economy in a setup in which ability and income are correlated with mortality. In such a situation, applying a uniform tax rate leads to reverse redistribution in the government pension system, since low income workers subsidize the pensions of high income workers, who have longer life expectancies. With the dramatic reduction in the tax rate required for a pay-as-you-go (paygo) pension under the Hybrid Reform, relative to the tax rate that is needed under Social Security, reverse redistribution is reduced, and therefore wealth inequality, relative to the Social Security equilibrium. Also, the presence of mandatory savings (PSA) implies that low income workers will

[2] Imrohoroğlu, Imrohoroğlu and Joines (1998a) discuss the main features of the model, used in many papers over the last twenty years.

accumulate wealth, and high income workers may substitute out some voluntary savings, thereby reducing wealth inequality further.

Related Literature. Our paper contributes to several strands of literature. First, it contributes to the literature on unfunded public pension systems. A significant part of this literature has revealed that the overall welfare effect of introducing such a system crucially depend on the importance of two opposing effects: a higher intergenerational risk sharing and a lower capital intensity. On the one hand, an unfunded social security system reallocates the impact of shocks across generations, reducing the consumption risk of the old aged relative to the risk they would face with private markets (Bohn 1999). This provides a welfare improvement for all generations alive and for the ones to be born in the future. On the other hand, such a system redistributes income away from younger agents with lower marginal propensities to consume, toward older agents with higher marginal propensities to consume. This lowers aggregate savings and aggregate capital formation (Feldstein 1974, Diamond 1977). The so-called "crowding-out" effect on capital from unfunded social security has been noted by many researchers over the years (see, for example, Auerbach and Kotlikoff 1987; Imrohoroğlu, Imrohoroğlu and Joines 1998b and 1999). This crowding out effect arises in life-cycle models, in which social security substitutes for precautionary savings to guard against an uncertain length of life. As in our setting, the net effect of lower capital intensity (due to crowding-out) is that agents would be better off in the long run if they were born into an economy without Social Security.[3]

Recently, Fuster, Imrohoroğlu and Imrohoroğlu (2005) also study the impact of mandatory Personal Security Accounts (PSA) in an economy with a dynastic framework. Under this assumption, the majority of households are better off with mandatory PSA than with paygo Social Security.[4] As in our paper, the authors find that wealth inequality increases when Social Security is introduced. In particular, low ability workers are disadvantaged by the mandatory PSA when it is not owned by them but merely funded during their working lifetime, while they are better off when the PSA is owned. One of the differences between our work and the one in Fuster et al (2005) is that they analyze a dynastic framework while we study a life cycle framework. Given this, we obtain a larger crowding-out effect in capital formation. In addition, we study a hybrid reform and we also analyze the impact of temptation preferences.

Related to the issue on inequality, much of the literature on Social Security has studied frameworks where mortality is independent of income. On the other hand, there is empirical evidence that low income is associated with high mortality rates and vice versa. Fuster (1999) and Fuster, Imrohoroğlu, Imrohoroğlu

[3] Subsequently, Fuster (1999) and Fuster, Imrohoroğlu and Imrohoroğlu (2002) have analyzed Social Security in a dynastic framework. In this setting, the household can undo the effect of Social Security through its altruistic motives, so that there is much less crowding-out.

[4] In Fuster et al (2005), the mandatory PSA was simply a mechanism to achieve funded Social Security. They also present results with an alternative lump-sum-at-retirement PSA, which is closer in spirit to the PSA of this paper. With the alternative PSA, the agent actually owns the PSA, and can bequeath it to heirs.

(2002, 2005) analyze a setup in which ability (and therefore income) is correlated with longevity. As we noted above, they find that Social Security increases wealth inequality. Gokhale et al (2001) also show that this correlation turns a paygo social security system into a "money pump", in the sense that it pumps wealth from working poor men to retired rich women. As discussed before, the present paper also finds that Social Security increases wealth inequality. With lower capital formation under Social Security, wages are depressed and capital returns are elevated. This hurts low income workers more than high income workers. Social Security also induces low income workers to reduce savings and bequests more significantly than do high income workers. However, capital formation is increased under the Hybrid Reform specified in this paper, thereby increasing wages and reducing interest rates. Furthermore, since workers own their PSA under the proposed reform, wealth increases directly for low income workers, so that, unlike Social Security, wealth inequality is reduced under the hybrid reform.

Second, our paper contributes to the literature on time inconsistency and self control. Typically, in much of the Social Security literature, the usual assumption is that households have standard preferences that exhibit a consumption smoothing motive. But it is empirically observed that preference-reversals can occur as time horizons change. Of two prizes in the distant future, the subject will choose the larger and later prize. But as the time to receive the prize draws closer, they would prefer the smaller but earlier prize. An excellent and fascinating survey of this literature is found in O'Donoghue and Rabin (1999).

Out of this literature two approaches towards modelling behavior with such preference reversals have arised. One approach is to model time inconsistent preferences with quasi-geometric discounting (Phelps and Pollak 1968; Laibson 1997). Imrohoroğlu, Imrohoroğlu and Joines (2003) analyzed Social Security under quasi-geometric (aka hyperbolic) discounting, and found that Social Security is a weak commitment device. The second approach to modelling behavior which exhibit preference reversals is to use temptation preferences (Gul and Pesendorfer 2001, 2004). Under temptation preferences, the agent has to exercise costly self-control to make the "right" choice. The choice can reverse if self-control becomes too costly. The advantage of this approach is that preferences are consistent over time, and can be formulated recursively. This naturally commends temptation preferences to the many problems which are approached with recursive models. A recent paper by Kumru and Thanopoulos (2007) used the Gul and Pesendorfer temptation preferences to study Social Security. They also find that Social Security is a commitment device that can increase welfare for agents afflicted with self-control problems.

This paper differs from Kumru and Thanopoulos in several important aspects. First, it extends the temptation preference analysis to agents which have heterogeneous ability and correlated mortality. Unlike Kumru and Thanopoulos (2007), this paper uses the formulation of Krusell, Kuruşçu and Smith (2005), assuming that an agent is tempted by a higher level of current consumption relative to future consumption, but is not tempted by changes to rankings of future consumption. Thus, agents would not necessarily be tempted to consume all their wealth currently. Instead, they would be tempted to shift some consumption to the present period, but otherwise leave future consumption rankings unchanged. In contrast, in the setup of Kumru and Thanopoulos, agents are tempted to consume all their wealth in the current period. Finally, this paper analyzes a hybrid Reform and finds it to be, unlike Social Security, an effective commitment device.

The paper is organized as follows. Section 2 presents the overlapping generations model which is used throughout this paper. The model is presented under standard expected utility preferences and temptation preferences. Section 3 discusses the calibration and solution method. Section 4 presents the results of the various experiments in both partial and general equilibrium and the welfare analysis. Finally, Section 6 summarizes and concludes.

2 The Overlapping Generations Model

The OLG model used in this paper is based on the 65-period model used by Imrohoroğlu, Imrohoroğlu and Joines (1997). It has been modified in two significant dimensions. First, the model has been extended to 85 periods to enable it to better capture advanced age dynamics. Second, we have introduced two types of workers, distinguished by their labor efficiency and correlated mortality rates, in order to analyze the effect that heterogeneous ability with correlated mortality has on a social security economy, and on alternative reform economies. Fuster, Imrohoroğlu and Imrohoroğlu (2002) also study a model with heterogeneous ability types, and this paper borrows from their setup. While their model is a dynastic setup, this paper uses a life-cycle model.

2.1 Demographics

Time is discrete, and each period represents one year. Age 1 corresponds to real age 21. The oldest possible age is age J , where $J = 85$ (real age 105). We assume that death is certain thereafter.

There are two types of agents indexed by z, where $z \in \{1, 2\}$. An agent's type is revealed at birth and this determines lifetime ability, which can be either high ($z = 1$) or low ($z = 2$). The realization of ability follows a first-order Markov process with transition matrix Π:

$$\begin{aligned} \Pi(z, z') &= [\pi_{ij}]; i, j \in \{1, 2\} \\ \pi_{ij} &= \Pr(z' = j | z = i). \end{aligned}$$

where z is the ability type of the parent and z' is the ability type of the child. It is assumed that the transition probabilities, π_{ij}, are such that there is a resulting stationary distribution of ability types, Λ, where $\lambda(z) \in \Lambda$, and $\lambda(1) + \lambda(2) = 1$.

The ability type determines the endowment of efficiency units an agent receives. In a given period, the cross-sectional labor efficiency $\varepsilon_j(z)$ is indexed by ability type z and age j. Without loss of generality, we assume throughout this paper that the rate of technological growth is zero. Under this assumption, the longitudinal efficiency units of a particular agent equal the cross-sectional efficiency factors, $\varepsilon_j^l(z) = \varepsilon_j(z)$.

Agents have uncertain lifetimes. Survival probabilities are correlated with ability type, so that high ability agents have longer expected lifetimes than low ability agents. Thus survival rates are indexed by age and type. The probability that an agent age j and ability type z survives to age $j+1$ is denoted by $\psi_j(z)$. The probability that an agent age j and ability type z survives to age $j+t$ is denoted by $\Psi_{j,t}(z)$, where:

$$\begin{aligned}\Psi_{j,t}(z) &= 1, \text{ if } t = 0 \\ \Psi_{j,t}(z) &= \Pi_{s=1}^{t} \psi_{j+s-1}(z), \text{ if } t > 0.\end{aligned}$$

Like much of the social security literature, this paper analyzes the steady states of a stationary population distribution, with time invariant cohort shares. Let ρ be the assumed constant rate of growth in population. Then, the cohort share of a new agent of type 1 relative to a new agent of type 2 is equal to $\frac{\lambda(1)}{\lambda(2)}$. That is, the size of the newborn type cohorts, relative to each other is determined by the Markov process stationary distribution of types. Thereafter, relative cohort shares are a result of the population growth rate and survival probabilities. Letting $\mu_j(z)$ denote the cohort share for an agent of age j and ability type z, and letting N denote the total population when the newborn cohort is indexed to one (1), for newborns ($j = 1$):

$$\begin{aligned}\mu_1(z) &= \lambda(z) \cdot [1/N], \text{ where} \\ N &= \sum_{z=1}^{2} \sum_{t=0}^{J} \lambda(z) \cdot (1+\rho)^{-t} \cdot \Psi_{1,t}(z).\end{aligned} \tag{1}$$

Each new cohort is $(1+\rho)$ times as large as the preceding cohort, and each cohort survives to the next period according to the corresponding age and ability type, $\psi_j(z)$. Thus for $j = 1, 2, ...J-1$:

$$\mu_{j+1}(z) = \mu_j(z) \cdot \frac{\psi_j(z)}{1+\rho}. \tag{2}$$

Finally, the sum of all cohorts must equal 100% so that,

$$\sum_{z=1}^{2} \sum_{j=1}^{J} \mu_j(z) = 1. \tag{3}$$

Given the Markov process, survival rates, and population growth rate, $\Pi, \psi_j(z)$ and ρ, respectively, the above relationships uniquely determine the time invariant cohort shares, $\{\mu_j(z)\}$.

2.2 Technology and Factor Prices

There is a single good in the economy, produced by one or more firms using a constant returns to scale Cobb-Douglas production function:

$$Y = AK^{1-\alpha} \cdot L^{\alpha}, \text{ where } \alpha \in (0, 1).$$

Total factor productivity A is normalized to 1. The labor share is α and K and L are aggregate capital and labor supplied as inputs. Capital is assumed to depreciate at the constant rate δ. Therefore, in a competitive equilibrium, we get factor prices for capital and labor:

$$\begin{aligned} r &= (1-\alpha) \cdot K^{-\alpha} \cdot L^{\alpha} - \delta \\ w &= \alpha \cdot K^{1-\alpha} \cdot L^{\alpha-1} \end{aligned} \tag{4}$$

K represents the aggregate asset holdings over the population in a given period. The size of L is determined by the workers up to retirement age j^*. Workers are assumed to supply labor inelastically to age j^*, and do not work thereafter. The actual supply of efficient labor depends on the ability type of agents in the working age population.

$$L = \sum_{z=1}^{2} \sum_{j=1}^{j^*-1} \varepsilon_j(z) \cdot \mu_j(z). \tag{5}$$

2.3 Government Policy and Social Security

This paper will analyze three model economies that differ in their approach to Government Policy and Social Security. There is a No Social Security economy (NSSE), a Social Security economy (SSE), and a Hybrid Reform economy (HRE).

2.3.1 No Social Security Model Economy

Government policy in the NSSE model is straightforward. There is no social security tax, and there are no social security benefits. Each worker must prepare for retirement income by saving during their working years, to build up a retirement nest egg.

2.3.2 Social Security Model Economy

In the SSE model, there is a social security program that provides a public pension to retirees. In this simplified model, the pension is a flat percentage θ of the average lifetime earnings. The social security benefit, denoted by b^{SS}, is given by:

$$b^{SS}(z) = \frac{\theta}{j^*-1} \cdot \sum_{j=1}^{j^*-1} w\varepsilon_j(z) \tag{6}$$

The role of the government is to collect a tax on labor income to exactly provide the social security pension to retirees. The necessary tax rate, τ_{SS}, in this pay-as-you-go model is:

$$\tau_{SS} = \frac{\sum_{z=1}^{2} \sum_{j=j^*}^{J} b^{SS}(z) \cdot \mu_j(z)}{w \cdot \{\sum_{z=1}^{2} \sum_{j=1}^{j^*-1} \varepsilon_j(z) \cdot \mu_j(z)\}} \tag{7}$$

The numerator is the total benefit paid under social security and the denominator is the total wage base over which the tax is applied in a given period.

2.3.3 Hybrid Reform Model Economy

The HRE model economy has a social security pension that starts at an advanced old age (AOA), i.e., later than the retirement age. Also, the HRE includes a mandatory personal security account (PSA), owned by the worker, in which mandatory contributions during the agent's working years are accumulated to provide a source for retirement income in the retirement years prior to the start of the public pension. The HRE model is called the Hybrid Reform economy, since the combination of personal accounts and an advanced old age public pension is a hybrid of the present system and a pure private accounts system.

The Hybrid Reform economy provides a public pension starting at age j^b, the benefits start age ($j^* < j^b < J$). The benefit amount is the same as in the social security economy. However, since benefits start at age j^b, rather than j^*, the tax to pay for the benefits is lower.

Let τ_{AOA} denote the tax rate needed to provide advanced old age benefits in the HRE. Then:

$$\tau_{AOA} = \frac{\sum_{z=1}^{2} \sum_{j=jb}^{J} b^{SS}(z) \cdot \mu_j(z)}{w \cdot \{\sum_{z=1}^{2} \sum_{j=1}^{j^*-1} \varepsilon_j(z) \cdot \mu_j(z)\}} \tag{8}$$

There is also a mandatory tax, τ_{PSA}, to accumulate a Personal Security Account (PSA). The PSA is a self-insurance mechanism which provides income for the years between retirement age, j^*, and the start of the public pension at advanced old age j^b.

There are many possible choices as to the specification of the self-insurance mechanism. We assume that contributions to the PSA are mandatory. It is a contribution of this paper that a mandatory funded pension is a superior commitment device to the present pay-as-you-go social security system. The level of funding is chosen so that the rate of contributions to the PSA plus the tax rate to provide the advanced old age public pension are the same in total as the tax rate to provide the public pension in the SSE model. Thus, $\tau_{SS} = \tau_{AOA} + \tau_{PSA}$.

It is assumed that contributions grow with interest equal to the return on capital r, and there are no taxes on that capital return. Let $a_j^{PSA}(z)$ denote the assets at the end of the period in a PSA owned by an agent of age j and type z. Define $a_0^{PSA}(z) = 0$ to be the initial balance in a newborn worker's PSA.Then during the agent's working years:

$$a_j^{PSA}(z) = \sum_{h=1}^{j} [\tau_{PSA} \cdot w \cdot \varepsilon_h(z)(1+r)^{j-h}], \text{ for } j < j^* \tag{9}$$

At retirement, the agent begins to deplete the PSA according to a pro-rata formula. In the event of death, the remaining PSA balance is bequeathed to the agent's heirs.[5] Each period from retirement age to the benefits start age, a pro-rata portion of the PSA is made available to the agent as income in that period. Let $b_j^{PSA}(z)$ denote the income benefit from the PSA to an agent of age j and type z. Then:

$$\begin{aligned} b_j^{PSA}(z) &= 0, \text{ for } j < j^* \\ b_j^{PSA}(z) &= \frac{a_{j-1}^{PSA}(z)}{j^b - j}, \text{ for } j = j^*, j^*+1, ...j^b - 1 \\ b_j^{PSA}(z) &= 0, \text{ for } j \geq j^b \end{aligned} \tag{10}$$

The remaining assets in the PSA accumulate with interest at rate r. Thus:

$$\begin{aligned} a_j^{PSA}(z) &= [a_{j-1}^{PSA}(z) - b_j^{PSA}(z)] \cdot (1+r), \text{ for } j = j^*, j^*+1, ...j^b - 1 \\ a_j^{PSA}(z) &= 0, \text{ for } j \geq j^b \end{aligned} \tag{11}$$

It should be recognized that $a_j^{PSA}(z) = 0$ in the SSE and NSSE models.

2.4 Constraints and Bequests

During the working years, the agent receives after-tax labor income based on their age/ability profile of labor efficiency. Upon retirement in the SSE model, the agent receives social security benefits. In the NSSE model, there are no social security benefits. In the HRE model, the retired agent receives distributions from the PSA each year from age j^* to age $j^b - 1$, and then receives a public pension from advanced old age j^b to the end of life.

Each period, the agent must choose the amount of consumption and the amount of voluntary saving. Savings earn the rate of return on capital r. We assume that agents are subject to a no borrowing constraint.

Because lifetimes are uncertain, many agents will die with positive amounts of assets (ak a accidental bequests). In the HRE model, agents younger than j^b will also have positive amounts of PSA assets. It is not uncommon for

[5] Fuster, Imrohoroğlu, and Imrohoroğlu study mandatory annuitization of a PSA which is not able to be bequeathed by the worker.

researchers to assume that these unintended bequests are confiscated and redistributed equally (per effective worker) to the survivors. This paper modifies that assumption somewhat. In this paper, accidental bequests are redistributed to surviving agents, in such a way that each type of agent receives an equal share based on the expected bequest of that agent, given their ability type. The amount of the bequest distributed to agents of ability type z is denoted by $\xi(z)$.

We can now describe the budget constraint faced by an agent of age j and ability type z, which is given by:

$$c_j(z) + a_{j+1}(z) = [a_j(z) + \xi(z)] \cdot (1+r) + Q_j(z) \tag{12}$$

The left hand side of the equation is the allocation of that period's wealth to consumption and savings, while the right hand side is the total of the resources available from prior savings and returns, bequests, wages and benefits (if any) from the social insurance mechanism. In all economies, $a_1(z) = 0$ and $a_{J+1}(z) = 0$. Further, for the three model economies, Q_j is defined as follows. In the No Social Security economy (NSSE):

$$\begin{aligned} Q_j(z) &= w \cdot \varepsilon_j(z) \ \text{ for } j < j^* \\ Q_j(z) &= 0 \ \text{ for } j^* \leq j \end{aligned}$$

In the Social Security economy (SSE):

$$\begin{aligned} Q_j(z) &= w \cdot \varepsilon_j(z) \cdot (1 - \tau_{SS}) \text{ for } j < j^* \\ Q_j(z) &= b^{SS}(z) \text{ for } j^* \leq j \end{aligned}$$

In the Hybrid Reform economy (HRE):

$$\begin{aligned} Q_j(z) &= w \cdot \varepsilon_j(z) \cdot (1 - \tau_{AOA} - \tau_{PSA}) \ \text{ for } j < j^* \\ Q_j(z) &= b_j^{PSA}(z) \cdot (1+r) \text{ for } j^* \leq j < j^b \\ Q_j(z) &= b^{SS}(z) \ \text{ for } j^b \leq j \end{aligned}$$

In all economies, households face a borrowing constraint:

$$a_j(z) \geq 0, \ \forall \ j$$

The bequest $\xi(z)$ is defined below.

2.4.1 Expected Bequests

Since ability type determines labor earnings, then the average size of an accidental bequest differs by type. It is reasonable to assume that children receive the accidental bequest left by the parent. The problem is how to allocate accidental bequests to agents of type 1 and type 2 so that the allocation is consistent with a presumption that the bequest stay in the family. To do this, let $Beq(z)$ denote the average bequest of agents of type z that die in a given period:

$$Beq(z) = \frac{\sum_{j=1}^{J}\{[a_j(z) + a_j^{PSA}(z)] \cdot \mu_j(z) \cdot (1 - \psi_j(z))\}}{\sum_{j=1}^{J} \mu_j(z) \cdot (1 - \psi_j(z))} \tag{13}$$

The numerator is the sum of assets owned by the type z agents who die in a given period, while the denominator is the number of such agents.

Recall that $\pi_{ij} \in \Pi$ is the probability that a parent of type i has a child of type j, and that $\lambda(z)$ is the probability that a newborn is type z. It turns out that the probability that a child of type z has a parent of a given type produces exactly the same Markov probability matrix Π. We use this fact to construct the ratio of the conditional expected bequest for agents of type z to the overall average bequest to agents of all types. We then use that ratio to allocate accidental bequests as follows:

$$\begin{aligned} \xi(z) &= \frac{\pi_{z1} \cdot Beq(1) + \pi_{z2} \cdot Beq(2)}{\sum_{z=1}^{2} \lambda(z) \cdot (\pi_{z1} \cdot Beq(1) + \pi_{z2} \cdot Beq(2))} \cdot Beq \qquad (14) \\ \text{where } Beq &= \sum_{z=1}^{2}\sum_{j=1}^{J}\{[a_j(z) + a_j^{PSA}(z)] \cdot \mu_j(z) \cdot (1 - \psi_j(z))\} \end{aligned}$$

Despite the extra complexity of this approach, there is a purpose in the specification of accidental bequests in such a way that it takes account of the likely ability type of the parent of the agent. That purpose is to build into the model, at least to this degree, the intergenerational effect of bequests under the alternative security regimes, when mortality is correlated with ability/income.[6]

2.5 Preferences and Individual Optimization Problem

A contribution of this paper is to analyze how the three model economies respond under temptation preferences, and to contrast those results to the results obtained under standard preferences. Standard preferences are defined below. These are followed by the specification of temptation preferences, as in the setup of Krusell, Kuruşçu and Smith, in which self-control over the interaction of normative and temptation utility produces overall utility. An expanded discussion of temptation preferences, as developed by Gul and Pesendorfer (2001), is presented in Appendix A.

To streamline notation, we let a denote $a_j(z)$, where age and type are defined by the context of the usage. Likewise, we will make the same notational shortcut for $\tilde{a}', c, \tilde{c}$, Q and ξ, where the tilda refers to the fact that the temptation choice for $\widetilde{c}$ and $\widetilde{a}'$ is from a different optimization than the normative choice c and a'. Also, we use the notational convention that the prime symbol denotes the next period value.

[6] In our experiments, the allocation of accidental bequests according to expectations, rather than equally across effective workers, did not result in significant differences. However, in possible extensions of this paper in which accidental bequests do give utility, this method of allocating bequests may prove to be more relevent to the outcome.

2.5.1 Standard Preferences

Standard preferences are defined over a lifetime sequence of consumption $\{c_j(z)\}_{j=1}^{J}$. The individual agent's objective for an agent age j is to maximize expected discounted lifetime utility:

$$U_j = \sum_{t=0}^{J-j} \beta^t [\Psi_{j,t}(z)] \cdot u(c_{j+t}(z)) \tag{15}$$

where $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, γ is the constant coefficient of relative risk aversion and β is the discount factor. The term in brackets is the probability of survival to age j for an agent of type z. Throughout this paper, these preferences are referred to as standard preferences.

An agent of age j and type z starts a given period with initial asset holdings a. The individual's dynamic problem is to choose how much to consume now c, and how much to save for future consumption a', in order to maximize the Bellman equation:

$$W_j(a) = \max_{c,a'} \{u(c) + \beta \psi_j(z) W_{j+1}(a')\} \tag{19}$$

subject to the budget constraint in (12), the borrowing constraint, the initial and optimality conditions, and taking the factor prices as given.

2.5.2 Temptation Preferences

Let U denote normative utility and V denote temptation utility. The normative utility is given by equation (15). Further, temptation utility is specified in terms of the felicity function u, and two parameters, a strength parameter σ and a future discount parameter φ:

$$V_j(\widetilde{c}, a, \widetilde{a}')) = \sigma[u(\widetilde{c}) + \varphi \beta \psi_j(z) W_{j+1}(\widetilde{a}')] \tag{17}$$

Putting this all together, an agent with temptation preferences maximizes:

$$\begin{aligned} W_j(a) &= \max_{c,a'} \{U_j(c, a, a') + V_j(c, a, a')\} - \max_{\widetilde{c},\widetilde{a}'} V_j(\widetilde{c}, a, \widetilde{a}')) \qquad (18) \\ &= \max_{c,a'} \{(1+\sigma)u(c) + (1+\sigma\varphi)\beta\psi_j(z)W_{j+1}(a')\} \\ &\quad - \max_{\widetilde{c},\widetilde{a}'} \sigma\{u(\widetilde{c}) + \varphi\beta\psi_j(z)W_{j+1}(\widetilde{a}')\} \end{aligned}$$

subject to the same conditions as under standard preferences. Note that c and $\widetilde{c}$ are given by:

$$\begin{aligned} c &= (a+\xi)\cdot(1+r) + Q - a' \\ \widetilde{c} &= (a+\xi)\cdot(1+r) + Q - \widetilde{a}' \end{aligned}$$

2.6 The Steady State Equilibrium

Let $D = \{d_1, d_2, ..., d_m\}$ represent the discrete set of values that asset holdings are permitted to take. The feasible set for an age j agent of type z and asset holdings a is denoted by $\Omega(j, a, z)$. The possible choices for a using standard preferences and for a and $\tilde{a}$ with temptation preferences satisfy $a' \in \Omega(j, a, z)$, $\tilde{a}' \in \Omega(j, a, z)$, $a' \geq 0$, $\tilde{a}' \geq 0$ and the budget constraints.

A steady state equilibrium for a set of policy parameters $\{\theta, \tau_{SS}, \tau_{AOA}, \tau_{PSA}\}$ is a collection of value functions $\{W_j(a)\}$; decision rules $R^c_{a,j,z} : D \times \{1, 2, ..., J\} \times \{1, 2\} \to \mathbb{R}_+$ and $R^{a'}_{a,j,z} : D \times \{1, 2, ..., J\} \times \{1, 2\} \to D$; a stationary distribution of types of newborns, $\{\lambda(1), \lambda(2)\}$; a time invariant distribution of agents by type, $\{\mu_j(z) | \forall j \in \{1, 2, ...J\}, \forall z \in \{1, 2\}\}$; a set of prices for capital and labor $\{r, w\}$; and a set of lump sum transfers of accidental bequests to agents $\{\xi(z)\}$; such that

1. Given factor prices, government policy and the lump sum transfers, the decision rules solve the individual optimization problem.

2. Factor prices solve the optimization problem of the firm

3. Markets clear, implying that:

$$K = \sum_{z=1}^{2} \sum_{j=1}^{J} [a_j(z) + \xi(z) + a^{PSA}_{j-1}(z)] \cdot \mu_j(z) \tag{21}$$

$$L = \sum_{z=1}^{2} \sum_{j=1}^{j^*-1} [\varepsilon_j(z) \cdot \mu_j(z)] \tag{22}$$

$$Y = \sum_{z=1}^{2} \sum_{j=1}^{J} \mu_j(z) \cdot [c_j(z) + a_{j+1}(z) + a^{PSA}_j(z) - (1-\delta) \cdot (a_j(z) + \xi(z) + a^{PSA}_{j-1}(z)\ )] \tag{23}$$

Thus, aggregate capital is the sum of individual asset holdings, aggregate labor is the sum of effective workers and output equals aggregate consumption plus the increase in aggregate capital.

4. The invariant population distribution conditions are satisfied.

5. Government pensions are fully financed by the labor tax.

6. The lump sum transfers satisfy:

$$\sum_{z=1}^{2} \sum_{j=1}^{J} \xi(z) \cdot \mu_j(z) = \sum_{z=1}^{2} \sum_{j=1}^{J} \{[a_j(z) + a^{PSA}_j(z)] \cdot \mu_j(z) \cdot (1 - \psi_j(z))\} \tag{24}$$

3 Calibration and Solution Method

3.1 Calibration

3.1.1 Demographic and Labor Parameters

The model is calibrated following some of the previous literature. Each period represents one year. The probabilities that make up the transition matrix Π are taken from the calibration used by Fuster, Imrohoroğlu and Imrohoroğlu (2002). As they explained, the values of Π were chosen to match 1991 Bureau of the Census data on the proportion of college graduates in the work force, and to match an observed correlation between the permanent component of income of parents and children, based on estimates by Zimmerman (1992) and Solon (1992). Thus:

$$\Pi = \begin{bmatrix} \pi(1,1) & \pi(1,2) \\ \pi(2,1) & \pi(2,2) \end{bmatrix} = \begin{bmatrix} .57 & .43 \\ .17 & .83 \end{bmatrix}$$

This transition matrix produces a stationary distribution of types for newborns $\Lambda = [\lambda(1)\ \lambda(2)] = [.2833\ .7167]$. Therefore, the proportion of newborns with high ability is 28.33% and the proportion of low ability is 71.67%.

Mortality is assumed to be different between high ability agents and low ability agents. The survival rates that are developed in three steps. First, the study begins with the same mortality rates used by Imrohoroğlu, Imrohoroğlu and Joines (1999) in their 65 period model, which is based on mortality rates from the Social Security Administration. The present paper extends the mortality rates 20 more periods, again based on mortality rates from the Social Security Administration. The aggregate mortality rates are then split into two sets of mortality rates; one for the high ability workers and one for the low ability workers. The method used to split aggregate mortality rates by type is explained in Appendix B.

The model also splits the aggregate labor efficiency factors into two sets, one for each type. The aggregate factors are the age-related factors used by Imrohoroğlu, Imrohoroğlu and Joines (1999), based on research by Hansen (1993). The aggregate efficiency factors are then split into type specific efficiency factors in a manner similar to that used to split the mortality rates. The details are explained in Appendix C. Labor is supplied inelastically to retirement age, which is fixed at $j^* = 45$, corresponding to real age 65. Thereafter, $\bar{\varepsilon}_j = 0$.

The population growth rate ρ is assumed to be a constant and equal to 1.2%, using the calibration by Fuster, Imrohoroğlu and Imrohoroğlu (2002). This corresponds to the average annual population growth rate of the United States over a fifty year period.

3.1.2 Technology Parameters

The model uses a Cobb-Douglas production function with constant returns to scale, $Y = AK^{1-\alpha}L^{\alpha}$. Total factor productivity A is normalized to one. The labor's share α is set to 0.64. These values are often used in a simple model, as they approximate the observed patterns in the US over a long period. The value of L is determined by the demographic assumptions.

Depreciation is set at a constant rate of 6.9%, following Imrohoroğlu, Imrohoroğlu and Joines (1999), in which they calculated their technology parameters based on annual US data since 1954. The rate of exogenous technological growth is set to zero.

3.1.3 Government Policy and Social Security

The replacement rate θ that is used to model social security is 40%, roughly comparable to the average level of benefits of Social Security. The age at which benefits start, j^b, is 45 (corresponding to real age 65) for the Social Security economy, and 60 (real age 80) for the Hybrid Reform economy.

3.1.4 Preferences

The parameters that specify standard preferences in the model are the coefficient of relative risk aversion, γ, and the time preference discount rate, β. The coefficient of relative risk aversion, γ, is set to 2, and the time preference discount rate, β, is set to .978. These values match the values used by Imrohoroğlu, Imrohoroğlu and Joines (1999), which were chosen to produce a capital intensity ratio close to 2.5, which is the empirical average in the US since 1954, according to their analysis. In this paper, under the SSE model, this parameterization produces a capital intensity of 2.9. While higher than the Imrohoroğlu et al model result, it is closer to the wealth output ratios reported by Cooley and Prescott (1995) and Laitner (1992), which were 3.32 and 3.15 respectively.

Temptation preferences are specified by the Temptation strength parameter, denoted by σ, and the temptation future discount factor, denoted by φ. Laibson Reppetto and Tobacman (1998) find that values for the hyperbolic discount factor in a model of hyperbolic discounting can be as low as .6, and estimate a value of 0.7. The temptation future discount factor is comparable to the hyperbolic discount factor. Imrohoroğlu, Imrohoroğlu and Joines (2000) use 0.85 and 0.9 for experiments with hyperbolic discount factors. This paper uses 0.7 and 0.9 as experimental values for the temptation future discount factor.

For the temptation strength parameter, this paper uses increasing degrees of temptation strength, with σ taking on the values 0.1, 0.2, and 0.3. [7]

3.1.5 Summary of Calibration and Resulting Tax Rates

Table 1 presents a summary of the calibration parameters.

[7] These will be referred to as weak, moderate and strong temptation respectively. As noted in the paper by Imrohoroğlu, Imrohorouglu and Joines (2000), in which they studied Social Security with a model using hyperbolic discounting in the utility function, as σ approached .3, their model became more difficult to work with. That was also the case in this paper. To find the equilibrium for the cases where $\sigma = .3$, it was necessary to increase the density of grid points in the state space by 67% over the density that was adequate for the cases where $\sigma = .1$. For consistency, all experiments were carried out at the higher grid density.

Table 1. Calibration Parameters

Demographics		
Population growth rate	ρ	.012
Maximum survival age	J	85 (real age 105)
Retirement age	j^*	45(real age 65)
Conditional survival probabilities	$\bar{\psi}(j)$	based on SSA Tables
Low ability mortality factor		112.6%
Efficiency profile	$\bar{\varepsilon}(j)$	Hansen (1993)
High ability efficiency factor		150%
Production		
Labor share	α	.64
Total factor productivity	A	1
Depreciation	δ	.069
Government Policy		
Benefits start age	j^*	45 (Social Security Economy)
	j^b	60 (Hybrid Reform Economy)
Preferences		
Relative risk aversion	γ	2
Discount rate	β	.978
Temptation strength parameter	σ	$(.1, .2, .3)$
temptation future discount rate	φ	various $(.9, .7)$

One of the distinguishing characteristics of the three alternative economic models is the tax rate which is needed to finance their respective social insurance programs. The rates are shown below. While technically a result of the model, they are a result of the invariant demographics and benefit design, not a result of economic behavior. Note that tax rates are unaffected by temptation preferences.

Table 2. Social InsuranceTax Rates

	NSSE	SSE	HRE
Tax to provide Paygo Pension	0	8.77%	2.11%
Tax to fund PSA	0	0	6.66%

3.2 Solution Method

The model is solved using a recursive method applied on a discretized state space. The solution we seek is a steady state of the economy. Starting from an initial guess as to the value of aggregate capital, K, and a guess as to the value of aggregate bequests, B, the solution algorithm computes optimal saving and consumption decisions for all agents in the invariant population distribution for a given period. New aggregate capital and aggregate bequests are calculated, given the optimal policies, and the new values for K and B are compared to the starting values. If they differ by more than a tolerance amount defined in advance, the starting guess is updated, and the algorithm is repeated. The process repeats until the aggregate capital and aggregate bequests reach a steady state.

To solve the individual problem under standard preferences, we can use a backwards induction algorithm, starting in the last period of life. To solve the problem with temptation preferences, consider the individual problem:

$$\begin{aligned} W_j(a) &= \max_{c,a'}\{(1+\sigma)u(c)+(1+\sigma\varphi)\beta\psi_j(z)W_{j+1}(a')\} \\ &\quad -\max_{\widetilde{c},\widetilde{a}'}\sigma\{u(\widetilde{c})+\varphi\beta\psi_j(z)W_{j+1}(\widetilde{a}')\} \end{aligned} \tag{20}$$

Since this problem involves two maximization problems, there will be two first order conditions needed. The first is the condition that leads to the solution of the first maximization term. This is the problem that determines the action the agent will take:

$$(1+\sigma)u'(c)=(1+\sigma\varphi)\beta\psi_j\frac{\partial}{\partial a'}[W_{j+1}(a')] \tag{26}$$

The second condition solves the second maximization problem, and it determines the disutility of self-control that will be expended by the agent. It affects his overall utility, but only indirectly the action chosen:

$$u'(\widetilde{c})=\varphi\beta\psi_j\frac{\partial}{\partial\widetilde{a}'}[W_{j+1}(\widetilde{a}')] \tag{27}$$

The solution to the problem is a set of decision rules, $g(a)$ and $\widetilde{g}(a)$ which determine the choices a' and $\widetilde{a}'$, respectively. This solution is also obtained using backward induction, solving the first order conditions in each period to find the optimal choice for a' and $\tilde{a}'$. The CRRA form of the felicity function makes the first order conditions analytically tractable. The details of the backward induction solution with temptation preferences and a CRRA felicity function are presented in Appendix D.

4 Numerical Results

4.1 Partial Equilibrium

Our first experiments analyze, in a partial equilibrium, the effect that each of the social insurance programs has on consumption (as a proxy for welfare), when introduced into a steady state economy with no social security. The social insurance programs are No Social Security, Social Security and Hybrid Reform. In the partial equilibrium analysis we make the important assumption that factor prices do not deviate from their steady state levels.

Table 3. Partial Equilibrium: Increase in Consumption

	SSE Overall	SSE High	SSE Low	HRE Overall	HRE High	HRE Low
Correlated	.29%	.59%	.05%	1.73%	1.93%	1.56%
Uncorrelated	.58%	0.50%	0.64%	1.90%	1.81%	1.97%

Starting from the NSSE steady state, we obtain the known result (Abel 1985) that a Social Security program can improve individual welfare when factor prices do not change. Consumption increases by .29% overall when Social Security is introduced into the economy. But even more dramatic is the 1.73% overall increase in consumption that results from the introduction of the Hybrid Reform program. What is behind this difference? In both the Social Security program and in the Hybrid Reform program, the worker pays a 2.11% payroll tax and receives a lifetime pension from age 80. The two programs differ with respect to the remaining 6.66% payroll tax, and pension benefits from age 65 to 80. In the case of Social Security, the additional 6.66% payroll tax is used to pay pension benefits to retirees between ages 65 and 80. In the case of the Hybrid Reform program, the 6.66% tax is accumulated in a PSA which is then depleted to provide an income benefit between age 65 and 80.

Evidently, based on the results in Table 3, potential consumption is reduced by using paygo taxes to provide pension benefits to younger aged retirees. We know from Samuelson (1959) and Abel (1985) that the rate of return on Social Security taxes will be the rate of population growth plus mortality gains from survivorship (tontine gains). But, in our world, in which workers routinely live to age 80, there are not meaningful tontine gains to be had prior to age 80. If, for clarity of argument's sake, we set the small pre-80 tontine gains to zero, then the worker in the SSE is giving up the opportunity to earn 3.33% (the equilibrium return under NSSE) in a PSA savings account, in return for a 1.2% (population growth rate) return on his paygo tax. This accounts for the lower consumption under SS than under the Hybrid Reform, in this partial equilibrium experiment.

The second finding from the partial equilibrium analysis is that in a world where mortality is correlated with ability/income, the benefits of social security accrue mainly to high income workers, who enjoy longer expected lifetimes. Table 3 breaks out the increase in consumption between high ability/high income worker and low ability/low income worker. It presents these results under the assumption that mortality is correlated with income, and also under the assumption that mortality is uncorrelated with income. When mortality is correlated with ability, Table 3 shows that almost all the welfare gain from introducing Social Security into a NSSE steady state economy accrues to the high income worker. In contrast, when mortality is not correlated to ability/income, the increase in consumption from adding social security is distributed quite evenly between the two classes of workers, even slightly favoring low income workers. The reason that welfare gains accrue to high income workers is the result of charging all workers a uniform tax rate, while low workers will receive fewer benefits due to shorter lives. This unwanted outcome is referred to as reverse redistribution in this paper.[8]

[8] In the real Social Security System, the benefit formula is skewed in favor of low income workers, which offsets to some degree the reverse subsidy for those workers who live to receive benefits. Historically, this was deemed to be a better solution than progressive tax rates, which might undermine public support for the whole system. Even with the skewed benefits, there is unremediated reverse redistribution.

When the Hybrid Reform is introduced, we see in Table 3 the same pattern of welfare gains by type of worker that we see with Social Security. If mortality is correlated with income, then high income workers benefit more than low income workers. If mortality is uncorrelated, then low income workers benefit relatively more than high income workers, from a reallocation of 6.6% payroll taxes to saving in a PSA. This pattern is to be expected, as the Hybrid Reform design is also subject to reverse redistribution. But since the Hybrid Reform pension covers a significantly smaller and later period of life, the tax is correspondingly low. Thus, while the paygo portion of the Hybrid Reform design means that reverse redistribution effects are present, they are small enough that investment gains in the PSA, which help low income workers relatively more than high income workers, and can offset some of the unequal effects. This is not the case with Social Security.

Third, we also find the well known result that, in a life cycle model, social security results in less capital formation than in a model without social security. The paygo pension is a substitute for precautionary savings (Barro 1974). Table 4 presents results which show this effect in a partial equilibrium, as workers make different choices about saving and consumption.

Table 4. Partial Equilibrium: Change in Key Indicators

	NSSE	SSE	HRE
% Change in Aggregate Savings	0%	-56.6%	-18.1%
% Change in Bequests	0%	-63.4%	-37.1%

Table 4 shows a dramatic reduction in savings when Social Security is introduced. Workers reduce their savings in response to the pension. But in the case of the Hybrid Reform, workers maintain most of their savings, reducing aggregate savings by only 18.1%, compared to 56.6% in the case of Social Security. This is not surprising, since they have to self fund retirement income prior to age 80.

But do extra savings add to individual welfare, or is it merely a case of dying richer? Table 4 gives insight into that question also. Starting from the NSSE steady state, we see that introducing a lifetime pension provision reduces bequests in both the SSE and HRE scenarios. In the SSE scenario, bequests (wealth at death) are reduced 63.4%, while aggregate savings (capital) are reduced 56.6%. On average, a 1% reduction in wealth at death in the SSE scenario corresponds to a .89% reduction in capital (i.e. $56.7 \div 63.4$). The same 1% reduction in wealth at death corresponds to only a .49% reduction in aggregate savings (capital) in the Hybrid Reform scenario ($18.1 \div 37.1$). In this sense, the Hybrid Reform is "bequest efficient", meaning that it is able to reduce wealth at death (which represents lost potential consumption) without reducing capital to the same degree. The reason for the bequest efficiency is simply that workers are highly likely to live to age 80, and realize (consume) the gains from self insuring, rather than dying prior to age 80 and leaving a bequest. This is a motivating insight into the self-insurance design of the Hybrid Reform.

In summary, our partial equilibrium analysis reveals that the Hybrid Reform is able to increase individual welfare relatively more than Social Security, when introduced into a steady state economy with no social security. It also reveals that the Hybrid Reform is able to distribute individual welfare gains more evenly by type of worker. And it shows the mechanism for these results to be closely tied to its efficiency with respect to converting savings into increased capital and consumption, not just into larger bequests. We now consider the Hybrid Reform in a general equilibrium setting, where factor prices adjust to changes in the economy brought by the alternative social insurance programs.

4.2 General Equilibrium Analysis

4.2.1 Steady State Equilibrium

In contrast to the partial equilibrium setting, in which factor prices are fixed, when factor prices are free to adjust to Social Security, the drop off in savings behavior in a standard preferences life-cycle model leads to lower capital, lower output and lower consumption per worker. A contribution of this paper is to examine general equilibrium outcomes when agents are subject to temptation preferences.

It is an empirical question as to what extent, if any, an economy is subject to issues of temptation and self-control. This paper takes an agnostic position and merely analyzes the impact of temptation preferences, without trying to quantify the presence of temptation and self-control problems. We want to know, assuming temptation preferences, what the impact on the equilibrium outcomes in the SSE, NSSE and HRE alternative economies will be.

Table 5 presents the equilibrium values for key economic indicators in the three model economies under standard preferences. The equilibrium values for key economic indicators for agents under moderate strength temptation preferences ($\sigma = .2$, $\varphi = .7$) are presented in Table 6.

Table 5. Steady State Values for Key Indicators (Standard Pref.)

		NSSE	SSE	HRE	SSE %NSSE	HRE %NSSE
Capital	K	5.7208	4.204	5.0913	73.5	89.0
Output	Y	1.6261	1.4554	1.5592	89.5	95.9
Consumption	C	1.1632	1.1152	1.1472	95.9	98.6
Welfare	W_1	-30.266	-33.960	-31.920		
Bequests	Beq	.11129	.07603	.08008	68.3	72.0
Wage	w	1.2986	1.1623	1.2452	89.5	95.9
Interest Rate	r	3.33%	5.56%	4.13%	167.0	124.0

Beyond the result that in general equilibrium, welfare in the long run (measured in terms of consumption) under Social Security is worse than under No Social Security, there are several other outcomes embedded in the results presented in Tables 5 and 6 which receive comment.

First, under standard preferences, Table 5 shows that capital, output and consumption in the HRE model are also worse than the corresponding outcomes in the NSSE model (89.0%, 95.9% and 98.6% respectively). However the crowding-out effect of social security on capital formation is not as severe in the HRE model (capital is 89.0% of NSSE vs. 73.5% in the SSE model). In other words, for the same total payroll tax, by allocating some of the payroll tax to a self-insured mandatory PSA, the HRE model economy maintains capital formation, leading to levels of consumption which are 98.6% of the NSSE result, as compared to 95.9% under SSE. The value function also indicates the welfare ranking among the three alternative models is consistent with the other indicators.

Table 6. Steady State Values for Key Indicators (moderate tempt.)

		NSSE	SSE	HRE	SSE %NSSE	HRE %NSSE
Capital	K	2.8134	2.3138	4.0101	82.2 %	142.5 %
Output	Y	1.2595	1.1739	1.4308	93.2	113.6
Consumption	C	1.0319	.98674	1.1063	95.6	107.2
Welfare	W_1	-35.712	-39.815	-33.639		
Bequests	Beq	.07017	.05221	.07072	74.4	100.8
Wage	w	1.0058	.93747	1.1427	93.2	113.6
Interest Rate	r	9.22%	11.36%	5.95%	123.2	64.5

Second, we see that temptation preferences have a strong negative impact on capital formation in the three model economies. Table 6 reflects that capital in the NSSE steady state is only 49% of the level of steady state capital under standard preferences. For the SSE and HRE model economies, the corresponding level of capital is 55% and 79% respectively. Agents are tempted to consume now, rather than save for future consumption. The SSE drop in capital is slightly less severe than the NSSE drop in capital, showing that Social Security is a commitment device, although a weak one.[9]

We also see that the ranking of the alternatives according to individual welfare is also different under temptation. The effectiveness of HRE as a commitment device is evident in that the value function for HRE is highest of the three under temptation.

Another important aspect reflected in Tables 5 and 6 is the bequest efficient design of the Hybrid Reform that was highlighted in the partial equilibrium setting. Under standard preferences, capital in the HRE steady state is 21.1% greater than in SSE, while bequests are only 5.3% greater. Under temptation the corresponding figures are 73.3% and 35.5%. A contribution of this paper is that there is an advantage to the macroeconomy and to individual welfare, from a bequest efficient Hybrid Reform.

Results from experiments under various levels of temptation are shown in the section on sensitivity analysis but one of the important findings is presented here.

[9] This result is consistent with a result obtained by Imrohoroğlu, Imrohoroğlu and Joines (2000), in their analysis of Social Security under hyperbolic discounting.

When we increase the degree of temptation, after an initial negative effect on capital formation, the mandatory PSA serves as an effective commitment device and insulates the HRE economy from further deterioration in capital, output, wages, and consumption. This is not the case for the SSE economy, which continues to see greater deterioration of capital at stronger levels of temptation. Table 7 illustrates the insulation effect of the Hybrid Reform compared to SSE.

Table 7. Effective Commitment Device?: Hybrid Reform vs. Social Security

	HRE Capital	HRE Consumption	SSE Capital	SSE Consumption
Standard Pref.	100 %	100 %	100 %	100 %
Weak Tempt.	80.4	96.8	70.9	93.7
Mod. Tempt.	78.8	96.4	55.0	88.5
Strong Tempt.	78.5	96.4	45.2	84.3

At low levels of temptation, the agent in the HRE economy is able to reduce voluntary saving for more consumption, and we notice the reduction in capital in Table 7 relative standard preferences. But at stronger levels of temptation, the agent is constrained from further reductions in saving by the mandatory PSA. This allows the economy to maintain capital, even if agents reduce voluntary savings to zero. This suggests that there is potentially significant value to a mandatory PSA, not only in increasing individual welfare (consumption), but also in immunizing the macroeconomy from adverse effects of temptation preferences on capital formation.

4.2.2 Individual Welfare, Reverse Redistribution and Wealth Inequality

In partial equilibrium, we saw that welfare gains under Social Security were captured by high income workers at the expense of low income workers. This same result is also evident in general equilibrium. Table 8 shows the change in individual welfare, relative to the welfare under the NSSE steady state. To show this change, we use the device of measuring the compensating variation; that is the amount of annual consumption a newborn would give up to be born into the NSSE economy, rather than the alternative economy.

Table 8. Individual Welfare by Type of Worker

	NSSE	NSSE	SSE	SSE	HRE	HRE
Standard Pref.	High	Low	High	Low	High	Low
Compensating Var.	0	0	10.0 %	12.8 %	4.3 %	5.8 %
Moderate Tempt.						
Compensating Var.	0	0	10.3 %	11.8 %	−5.0 %	−6.0 %

There are a couple observations from this table. First, reverse redistribution in SSE model persists in general equilibrium with standard preferences and with temptation preferences. Under standard preferences, high income workers in SSE have compensating variation of 10.0%, while low income workers have a compensating variation of 12.8%; 28% greater. Under temptation preferences, the differences between high income and low income welfare, relative to standard preferences, persist. One might have speculated that under temptation low income workers in SSE would not be as adversely affected, due to the commitment device of Social Security. And this appears to be the case, since low income workers compensating variation is now only 14.6% greater than high income worker's.

In the HRE model under standard preferences, we see that low income workers would pay up to 5.8% of annual consumption to be born into the NSSE economy, but that under temptation preferences, they would want to be paid 6.0% of annual consumption to be born into the NSSE economy. A similar outcome also holds for high income workers. Clearly, HRE is an effective commitment device against temptation.

Second, in the HRE model, the reverse redistribution that is evident under standard preferences disappears when temptation is taken into the model. Under temptation, the high income worker is more adversely affected, and the low income worker is relatively better off. It is certainly the case that the high income worker under standard preferences would have greater voluntary savings. Since voluntary savings are likely to be consumed under temptation, the high income worker would be adversely affected more than low income workers who would have little voluntary savings to consume. Furthermore, the scarcity of capital will produce a higher return on the PSA, which benefits both types of workers proportionately. So we see a reversal of positions under temptation. This is not to say that the reverse subsidy is gone. It is still present, inherent in the single tax rate design for a public pension. But the impact of the reverse subsidy is overwhelmed by the adverse effect of temptation, which hits high income workers harder than low income workers.

Table 10. Wealth Inequality in the Alternative Economies

Standard Preferences	NSSE	SSE	HRE
Wealth Gini Coefficient	.5481	.5116	.4812
% Wealth Owned by High Ability	48.8%	50.8%	48.5%
% Wealth Owned by High Ability	51.2%	49.2%	51.5%
Moderate Temptation			
Wealth Gini Coefficient	.6257	.5721	.4811
% Wealth Owned by High Ability	49.7%	50.6%	45.0%
% Wealth Owned by High Ability	50.3%	49.4%	55.0%

We observe in Table 10 that the wealth Gini coefficient indicates wealth inequality is improved in the SSE steady state over the NSSE steady state, and it improves even more in the HRE steady state. In both cases, the improvement arises from the result that a paygo pension improves the wealth of poor elderly households.

When wealth is aggregated across ages by type, we see a somewhat different story. In the NSSE economy, high ability agents own 48.8% of the aggregate wealth, while low ability workers own 51.2% of the wealth. (All the economies have the same invariant population distribution, in which 29.7% of the population are high ability and 70.3% are low ability). Under SSE, high ability agents increase their ownership of wealth to 50.8%, while low ability agents drop to 49.2%. This appears to contradict the finding of the Gini coefficient. But the two outcomes are consistent, taking into account that the poorest households are largely elderly, who are disproportionately high ability since they have a life expectancy which is five years longer than low ability workers.

Under temptation, wealth inequality is increased even further, except under HRE, where wealth inequality is decreased. Temptation has a bigger impact on high income agents, who have more available resources to consume. Temptation raises interest rates in all economies, as capital becomes more scarce. The low income worker in HRE is able to benefit from the higher returns, because of the mandatory PSA. A significant outcome is that under the HRE, not only does the Gini coefficient demonstrate improved wealth distribution, but there is improvement in the share of wealth owned by low ability agents over both the NSSE model and the SSE model.

Our findings from the general equilibrium analysis can be summarized as follows. (i) Under standard preferences, NSSE produces the highest levels of capital, output and consumption. (ii) When agents have temptation preferences, both NSSE and SSE economies suffer significant reductions in capital, output and consumption. Only the HRE is able to sustain the macroeconomy under temptation, as the mandatory PSA is an effective commitment device. (iii) When mortality is correlated with income, then Social Security increases wealth inequality as the rich capture much of the benefit of social security. However, wealth inequality is reduced in HRE even below the levels of NSSE economy. This is especially apparent under temptation preferences. (iv) The Hybrid Reform under standard preferences achieves capital formation levels close to that of NSSE, and under temptation it exceeds the capital formation of the NSSE model economy. The HRE also avoids most of the problem of reverse redistribution by means of the self-insurance mechanism, while it retains the social welfare benefits of a government guaranteed income at the advanced ages of life.

5 Conclusions

Even under standard preferences, there is a good case to be made for the second best welfare alternative for the long run, a Hybrid Reform proposal. As we have seen, the Hybrid Reform imposes a mandatory funding mechanism which improves capital formation in the economy. Ownership of the PSA is in the hands of the worker, which eliminates reverse redistribution with respect to the mandatory funding portion of the payroll tax. The HRE combination of funded self-insurance for early retirement benefits, and an unfunded public pension for advanced old age provides for greater capital formation, with better equity between workers of different income levels, while still providing an old age pension and avoiding the free rider problem inherent in NSSE.

But it is under temptation preferences that the Hybrid Reform really outperforms either of the two other model economies. In both the SSE and NSSE economies, temptation preferences produce a significant depressive effect on capital formation, output, and consumption. While SSE was slightly less sensitive to temptation than the NSSE economy, their differences were overwhelmed by their similarity in response to temptation preferences.

Unlike the NSSE and SSE economies, the HRE model economy contains an effective commitment mechanism, the mandatory PSA, which immunizes capital formation, output and consumption from the worst effects of temptation preferences. As a result, the long run welfare analysis showed advantages to workers in the HRE economy, while also providing insulation to the macroeconomy from the effects of temptation. Harnessing the dual power of a government provided pension at advanced old ages, and a self-insured mandatory PSA during younger retirement ages, the HRE is able to restore capital formation and immunize the economy from the deleterious effects of temptation preferences on voluntary savings.

As much of the literature, the present work has provided a steady state analysis and the results should therefore be interpreted with care. Clearly, it would be interesting to analyze the transition path to establish conclusive results. This is a very important issue that we leave for further research.

6 Appendix

Appendix A. Temptation Preferences

This paper analyzes the three model economies using temptation preferences, as presented by Gul and Pesendorfer in a series of articles in 2001, 2002, and 2004. The axiomatic development of a utility function that represents temptation preferences is presented in Gul and Pesendorfer (2001). Below is a summary of the intuition of temptation preferences.

The setting consists of an individual representative agent who will choose today a set of alternatives B, among which she will choose a consumption lottery next period. Assuming that the agent is an expected utility maximizer, the next period the agent chooses lottery $p \in B$ which solves $\max_{p \in B} \int u(p)dp$. At time 0, our agent prefers the set of alternatives B to the set of alternatives B' when $\max_{p \in B} \int u(p)dp \geq \max_{p \in B'} \int u(p)dp$. In the theory of expected utility, a standard axiom is that $B \succsim B' \Rightarrow B \sim B \cup B'$ (Kreps (1979)). This axiom rules out situations where the agent may benefit from or be harmed by the addition of alternatives in B'. The idea of temptation preferences is that the agent may strictly prefer B to $B \cup B'$, knowing that B' contains temptations to which she will be vulnerable in time 1. Thus, in the Gul and Pesendorfer development of temptation preferences, the standard axiom is relaxed to the 'set betweenness' axiom:

$$B \succsim B' \Rightarrow B \succsim B \cup B' \succsim B'$$

With the 'set betweenness' axiom, if $B \succsim B \cup B'$ it is possible for the agent at time 0 to strictly prefer the smaller set B of time 1 choices, to the larger set $B \cup B'$ which contains tempting choices that the agent would rather not face at time 1. The idea here is that the presence of temptations at time 1 are the reason for the preference for commitment to a choice set at time 0. To avoid the temptation, at time 0 our agent strictly prefers the set of alternatives B. In fact, this is the criteria by which a temptation is identified.

As Gul and Pesendorfer (2001) show, the set betweenness axiom together with the other axioms that otherwise lead to standard expected utility, lead instead to the utility representation of temptation preferences. The utility representation is as follows. Let $u(p)$ and $v(p)$ be von Neumann-Morgenstern expected utility functions. Intuitively, u is the utility function that is not affected by temptation, and v is the utility function that is affected by temptation. Then temptation preferences are represented by the function:

$$U(B) = \max_{p \in B} \int (u(p) + v(p))dp - \max_{\widetilde{p}' \in B} \int v(\widetilde{p})d\widetilde{p}$$

Note that when the agent has only one choice, then the function u is the agent's utility; the v terms net to zero. Thus Gul and Pesendorfer refer to $u(p)$ as 'the commitment utility', since it represents utility when one is committed to p (i.e., $B = \{p\}$). It is worth noting here that an agent with perfect foresight would also have utility u, and the standard assumption $B \sim B \cup B'$ would apply. An agent with perfect foresight is not vulnerable to temptation; for such a decision-maker $v = u$ and therefore the utility formulation above collapses to the usual expected utility function. Thus other authors refer to u as 'normative' utility, which is the terminology used in this paper.

The utility function v represents the temptation utility. In the model presented by Gul and Pesendorfer (2002), the agent maximizes $u + v$, arriving at the optimal compromise, with the expenditure of self-control with disutility

$$v(c) - \max_{\widetilde{p} \in B} \int v(\widetilde{p})d\widetilde{p}$$

where c is $\arg\max_{p \in B} \int (u(p) + v(p))dp$.

As Gul and Pesendorfer (2002) point out, temptation is closely related to the literature on time-inconsistent preferences. Time-inconsistent preferences lead to a solution strategy in which multiple selves are players in a game to find an equilibrium which is consistent. Krusell, Kuruşçu and Smith (2005) show that the Phelps-Pollack form of time-inconsistent preferences are equivalent to temptation preferences under a setup where the strength of the temptation goes to infinity.

An important feature of temptation preferences, and the reason they are useful in this analysis of Social Security, is that unlike the Phelps-Pollack formulation of time inconsistent preferences, temptation preferences are amenable to a dynamic setting. In the finite horizon case, the setting is that an agent chooses a consumption lottery from a set of alternatives B_t, and chooses a decision problem set B_{t+1} for the next period. Formally,

$$W_{t-1}(B_t) = \max_{p \in B_t} \int [u_t(p) + W_t(p, B_{t+1}) + V_t(p, B_{t+1})] dp - \max_{\widetilde{p} \in B_t} \int V_t(\widetilde{p}, B_{t+1}) d\widetilde{p}$$

In this representation, W_{t-1} represents the value function of the agent over the period t choices; prior to period t, the agent is committed to choose from among the choices in B_t. The normative (commitment) utility is $u_t + W_t$, and the temptation utility is V_t. In the final period (period T), the decision problem set is a singleton set, typically $\{0\}$ or the empty set (i.e., $B_{T+1} = \{0\}$).

$$W_{T-1}(B_T) = \max_{p \in B_T} \int [u_T(p) + 0 + V_T(p)] dp - \max_{\widetilde{p} \in B_T} \int V_T(\widetilde{p}) d\widetilde{p}$$

Following the approach of Krusell, Kuruşçu and Smith (2005), this paper uses a finite horizon, discrete time dynamic setting for temptation preferences, as shown in the description of the model below.

Appendix B. Method to Split Aggregate Mortality Rates by Type

The rates are split as follows. Low ability workers are assumed to have mortality rates that are a constant multiple of the aggregate mortality rate. Let x denote this constant multiple ($x > 1$). Let $\bar{\psi}_j$ denote the aggregate survival rate at age j, based on the Social Security Administration table. Then $(1 - \psi_j(2)) = x(1 - \bar{\psi}_j)$. We then solve for $\psi_j(1)$ so that the aggregate rate $\bar{\psi}(j)$ equals the weighted average of $\psi_j(1)$ and $\psi_j(2)$. The weights used are the age dependent proportion of the cohort by type. Since the mortality rates differ there is a gradually increasing proportion of higher ability agents as the cohort ages. We develop $\psi_j(1)$ recursively, starting with $j = 1$:

For $j = 1$:

$$\begin{aligned} 1 - \psi_1(2) &= x(1 - \bar{\psi}_1) \\ 1 - \psi_1(1) &= \frac{1 - \bar{\psi}_1 - p_2 \cdot [1 - \psi_1(2)]}{p_1} \end{aligned}$$

where

$$\begin{aligned} p_1 &= \frac{\lambda(1)}{\lambda(1) + \lambda(2)} \\ p_2 &= \frac{\lambda(2)}{\lambda(1) + \lambda(2)} \end{aligned}$$

For $j = 2$:

$$\begin{aligned} 1-\psi_2(2) &= x(1-\bar{\psi}_2) \\ 1-\psi_2(1) &= \frac{1-\bar{\psi}_2 - p_2 \cdot [1-\psi_2(2)]}{p_1} \end{aligned}$$

where

$$\begin{aligned} p_1 &= \frac{\lambda(1)\cdot\Psi_{1,j-1}(1)}{\lambda(1)\cdot\Psi_{1,j-1}(1)+\lambda(2)\cdot\Psi_{1,j-1}(2)} \\ p_2 &= \frac{\lambda(2)\cdot\Psi_{1,j-1}(2)}{\lambda(1)\cdot\Psi_{1,j-1}(1)+\lambda(2)\cdot\Psi_{1,j-1}(2)} \end{aligned}$$

For a general j until $j = J = 85$, where $\psi_J(z) = 0$:

$$\begin{aligned} 1-\psi_j(2) &= x(1-\bar{\psi}_j) \\ 1-\psi_j(1) &= \frac{1-\bar{\psi}_j - p_2 \cdot [1-\psi_j(2)]}{p_1} \end{aligned}$$

where

$$\begin{aligned} p_1 &= \frac{\lambda(1)\cdot\Psi_{1,j-1}(1)}{\lambda(1)\cdot\Psi_{1,j-1}(1)+\lambda(2)\cdot\Psi_{1,j-1}(2)} \\ p_2 &= \frac{\lambda(2)\cdot\Psi_{1,j-1}(2)}{\lambda(1)\cdot\Psi_{1,j-1}(1)+\lambda(2)\cdot\Psi_{1,j-1}(2)} \end{aligned}$$

With this method of splitting the aggregate mortality rate, x is chosen so that the expected lifetime of high ability workers (college graduates represent high ability workers) is five years longer than the expected lifetime of low ability workers. This same five year differential was used by Fuster, Imrohoroğlu and Imrohorglu (2002), based on research by Elo and Preston (1996), who found a five year differential in life expectancies between college graduates and non-graduates. It can be found that $x = 1.126$ produces the desired outcome. The model results in a life expectancy of the high ability group that corresponds to real expected life of 76.1 years, and for the low ability group a life expectancy of 71.1.

Appendix C. Method to Split the Efficiency Factors by Type

First, based on data from the Bureau of the Census and the Bureau of Labor Statistics, the model assumes that the high ability group (college graduates) earns 150% of the aggregate wage. Thus, if $\bar{\varepsilon}_j$ denotes the aggregate labor efficiency factor at age j, then $\varepsilon_j(1) = 1.5 \cdot \bar{\varepsilon}_j$ for $j < j^*$. Efficiency factors for low ability workers are then determined so that the weighted average of the two types match the aggregate factor at each age. The weights p_1 and p_2 (i.e. the proportion of the surviving cohort at a given age j that is type 1 and 2 , respectively), which are used to reproduce the average efficiency factor, are the same weights used earlier when splitting the aggregate mortality rates. For $j = 1, 2, ...j^* - 1$:

$$
\begin{aligned}
\varepsilon_j(1) &= 1.5 \cdot \bar{\varepsilon}_j \\
\varepsilon_j(2) &= \frac{\bar{\varepsilon}_j - p_1 \cdot \varepsilon_j(1)}{p_2}
\end{aligned}
$$

where

$$
\begin{aligned}
p_1 &= \frac{\lambda(1) \cdot \Psi_{1,j-1}(1)}{\lambda(1) \cdot \Psi_{1,j-1}(1) + \lambda(2) \cdot \Psi_{1,j-1}(2)} \\
p_2 &= \frac{\lambda(2) \cdot \Psi_{1,j-1}(2)}{\lambda(1) \cdot \Psi_{1,j-1}(1) + \lambda(2) \cdot \Psi_{1,j-1}(2)}
\end{aligned}
$$

Appendix D. Solution Method

The agent is indexed at the start of each period according to age, asset holdings, and type, (j, a, i). Age is naturally discretized by $j \in \{1, 2, ...J\}$. Type is either high ability ($z = 1$) or low ability ($z = 2$). It remains to discretize the state space for asset holdings. Let $D = \{d_1, d_2, ..., d_m\}$ represent the discrete set of values that asset holdings are permitted to take. With the calibration described here, a reasonable corresponding grid of values for possible asset holdings is on the interval $[0, 32]$. To get a sufficient smoothness without adding computational time, the model uses a greater concentration of grid points on the interval $[0, 12]$. This analysis uses 6001 uniform grid points for asset holdings on the interval $[0, 12]$, and 1400 uniform grid points for the interval $(12, 32]$.

The feasible set for an age j agent of type z and asset holdings a is denoted by $\Omega(j, a, z)$. The possible choices, a in the experiments using standard preferences, and a and $\tilde{a}$ for temptation preferences, satisfy $a' \in \Omega(j, a, z)$, $\tilde{a}' \in \Omega(j, a, z)$, $a' \geq 0$, $\tilde{a}' \geq 0$, and the budget constraints are satisfied.

The details of the backward induction solution with Temptation preferences and a CRRA felicity function follow.

The Last Period. Starting with the last period, the problem is greatly simplified by the assumption that there is no utility from bequests. The optimal decision for both normative and temptation utility is therefore to consume everything. The value function is therefore:

$$
\begin{aligned}
W_J(a) &= u(c_J) \\
\text{where } c_J &= (a + \xi_J)(1 + r) + Q_J
\end{aligned}
$$

Anticipating the soon-to-be-needed derivative, we note that $\frac{\partial}{\partial a}[W_J(a)] = u'(c_T)(1 + r)$.

The Next to Last Period. In the next to last period, the value function is:

$$\begin{aligned} W_{J-1}(a) &= \max_{c,a}\{(1+\sigma)u(c_{J-1}) + (1+\sigma\varphi)\beta\psi_{J-1}(z)W_J(a')\} \\ &\quad - \max_{\widetilde{c},\widetilde{a}'}\sigma\{u(\widetilde{c}_{J-1}) + \varphi\beta\psi_{J-1}(z)W_J(\widetilde{a}')\} \\ \text{where } c_{J-1} &= (a+\xi_{J-1})(1+r) + Q_{J-1} - a' \\ \widetilde{c}_{J-1} &= (a+\xi_{J-1})(1+r) + Q_{J-1} - \widetilde{a}' \end{aligned}$$

There are two first order conditions:

$$(1+\sigma)u'(c_{J-1}) = (1+\sigma\varphi)\beta\psi_{J-1}\frac{\partial}{\partial a'}[W_J(a')]$$

$$u'(\widetilde{c}_{J-1}) = \varphi\beta\psi_{J-1}\frac{\partial}{\partial \widetilde{a}'}[W_J(\widetilde{a}')]$$

Substituting for the derivative of the last period value function, we get the following Euler equations:

$$(1+\sigma)u'(c_{J-1}) = (1+\sigma\varphi)\beta\psi_{J-1}u'(c_J)(1+r)$$

$$u'(\widetilde{c}_{J-1}) = \varphi\beta\psi_{J-1}u'(\widetilde{c}_J)(1+r)$$

In Euler equation 1, c_J is a function of a_J. But $a_J = a' = g(a_{J-1})$, so then c_J is a function of a'. Note that c_{J-1} is also a function of a'. So the Euler equation itself is an implicit function of a'. Likewise, in Euler equation 2, $\widetilde{c}_J$ is a function of $\widetilde{a}' = \widetilde{g}(a_{J-1})$, and $\widetilde{c}_{J-1}$ is also a function of $\widetilde{a}'$. So Euler equation two is an implicit function of $\widetilde{a}'$. Thanks to the mathematically tractable nature of the CRRA form of the felicity function, we can solve these equations for a'. Using the CRRA form for $u(c)$, Euler equation 1 becomes:

$$(1+\sigma)\cdot(c_{J-1})^{-\gamma} = (1+\sigma\varphi)\beta\psi_{J-1}\cdot(c_J)^{-\gamma}\cdot(1+r)$$

Substituting for c_{J-1} and for c_J we get:

$$\begin{aligned} (a_{J-1}+\xi_{J-1})(1+r) + Q_{J-1} - a' &= [\frac{(1+\sigma\varphi)}{(1+\sigma)}\beta\psi_{J-1}(1+r)]^{\frac{-1}{\gamma}}\cdot ... \\ &\quad ((a'+\xi_J)(1+r) + Q_J) \end{aligned}$$

Likewise for Euler equation 2:

$$\begin{aligned} (a_{J-1}+\xi_{J-1})(1+r) + Q_{J-1} - \widetilde{a}' &= [\sigma\varphi\beta\psi_{J-1}(1+r)]^{\frac{-1}{\gamma}}... \\ &\quad \cdot((\widetilde{a}'+\xi_J)(1+r) + Q_J) \end{aligned}$$

Given the state variable a, the two Euler equations can be solved for a' and $\widetilde{a}'$, respectively. (Note that Q is not dependent on a.) We thus get solutions for

$g(a) = a'$, and for $\widetilde{g}(a) = \widetilde{a}'$. Again anticipating the need for the derivative of the value function, we get:

$$\begin{aligned}
\frac{\partial}{\partial a}[W_{J-1}(a)] &= (1+\sigma) \cdot u'(c_{J-1})(1+r) - \sigma u'(\widetilde{c}_{J-1})(1+r) \\
\text{where } c_{J-1} &= (a+\xi)(1+r) + Q_{J-1} - g(a) \\
\widetilde{c}_{J-1} &= (a+\xi)(1+r) + Q_{J-1} - \widetilde{g}(a)
\end{aligned}$$

The Next to Next to Last Period. Moving backwards to the next to next to last period, the value function is:

$$\begin{aligned}
W_{J-2}(a) &= \max_{c,a}\{(1+\sigma)u(c_{J-2}) + (1+\sigma\varphi)\beta\psi_{J-2}W_{J-1}(a')\}... \\
&\quad - \max_{\widetilde{c},\widetilde{a}'} \sigma\{u(\widetilde{c}_{J-2}) + \varphi\beta\psi_{J-2}W_{J-1}(\widetilde{a}')\}
\end{aligned}$$

$$\begin{aligned}
\text{where } c_{J-2} &= (a+\xi)(1+r) + Q_{J-2} - a' \\
\widetilde{c}_{J-2} &= (a+\xi)(1+r) + Q_{J-2} - \widetilde{a}'
\end{aligned}$$

There are two first order conditions. Euler equation 1 is:

$$\begin{aligned}
(1+\sigma)u'(c_{J-2}) &= (1+\sigma\varphi)\beta\psi_{J-2}\frac{\partial}{\partial a'}[W_{J-1}(a')] \Rightarrow \\
(1+\sigma)u'(c_{J-2}) &= (1+\sigma\varphi)\beta\psi_{J-2} \cdot (1+\sigma) \cdot u'(c_{J-1})(1+r) - \sigma u'(\widetilde{c}_{J-1})(1+r) \\
\text{where } c_{J-1} &= (a'+\xi)(1+r) + Q_{J-1} - g(a') \\
\widetilde{c}_{J-1} &= (a'+\xi)(1+r) + Q_{J-1} - \widetilde{g}(a')
\end{aligned}$$

Euler equation 2 is:

$$\begin{aligned}
u'(\widetilde{c}_{J-2}) &= \varphi\beta\psi_{J-2}\frac{\partial}{\partial \widetilde{a}'}[W_{J-1}(\widetilde{a}')] \Rightarrow \\
u'(\widetilde{c}_{J-2}) &= \varphi\beta\psi_{J-2} \cdot (1+\sigma) \cdot u'(\widehat{c}_{J-1})(1+r) - \sigma u'(\widetilde{\widetilde{c}}_{J-1})(1+r) \\
\text{where } \widehat{c}_{J-1} &= (\widetilde{a}'+\xi)(1+r) + Q_{J-1} - g(\widetilde{a}') \\
\widetilde{\widetilde{c}}_{J-1} &= (\widetilde{a}'+\xi)(1+r) + Q_{J-1} - \widetilde{g}(\widetilde{a}')
\end{aligned}$$

As we did in the next to last period, we substitute for the derivative of the next period value function, and use the CRRA form of the felicity to arrive at a solvable set of Euler equations.

$$\begin{aligned}
(a_{J-2}+\xi_{J-2})(1+r) + Q_{J-2} - a' &= [\frac{(1+\sigma\varphi)}{(1+\sigma)}\beta\psi_{J-2}(1+r)]^{\frac{-1}{\gamma}}... \\
&\quad \cdot((a'+\xi_{J-1})(1+r) + Q_{J-1} - g(a'))
\end{aligned}$$

$$\begin{aligned}
(a_{J-2}+\xi_{J-2})(1+r) + Q_{J-2} - \widetilde{a}' &= [\sigma\varphi\beta\psi_{J-2}(1+r)]^{\frac{-1}{\gamma}}... \\
&\quad \cdot((\widetilde{a}'+\xi_{J-1})(1+r) + Q_{J-1}) - \widetilde{g}(\widetilde{a}')
\end{aligned}$$

Exactly as before, Euler equation 1 is an implicit function of a', . Likewise, Euler equation two is an implicit function of $\widetilde{a}'$. Again, thanks to the mathematically tractable nature of the CRRA form of the felicity function, we can solve these equations for a' and $\widetilde{a}'$.

The problem is solved by continuing to work backwards, solving the Euler equations for the decision rules at each period in the same manner as above. Since our model is a discrete model, it chooses the value $d_i \in D$ closest to the analytical values for a' and $\widetilde{a}'$.

Given the initial guess as to the level of aggregate capita, K,and aggregate bequests, B, in the model economy, the model solves the individual's dynamic problem using the backwards induction algorithm. The optimal decision rules for consumption and saving result in a particular computed value for aggregate capital and aggregate bequests in the economy at the end of the period. If the ending values for K' and B', matches the starting values, then the economy is in a steady state. If not, the solution algorithm iterates with updated starting values for K and B. With standard preferences, we are assured of a steady state solution, since the update algorithm is constructed so that Blackwell's sufficiency conditions are satisfied. While the Temptation preferences do not satisfy Blackwell's sufficiency conditions, we proceed carefully along this algorithm and reach a steady state nonetheless.

Appendix E. Sensitivity to Temptation Parameters

Table 11. Key Indicators under Temptation (σ, φ)

	St. Pref.	(.1, .9)	(.2, .9)	(.3, .9)	(.1, .7)	(.2, .7)	(.3, .7)
Capital							
NSSE	5.720	4.9764	4.4323	4.0593	3.8296	2.8134	2.235
SSE	4.204	3.7214	3.3731	3.1338	2.9801	2.3138	1.898
HRE	5.091	4.5713	4.2674	4.1331	4.0959	4.0101	3.996
Welfare							
NSSE	−30.25	−31.05	−31.82	−32.48	−32.96	−35.71	−38.15
SSE	−33.96	−34.95	−35.82	−36.52	−37.01	−39.81	−42.31
HRE	−31.92	−32.68	−33.14	−33.30	−33.40	−33.63	−33.73
Comp. Var.	High in.						
NSSE	0	0	0	0	0	0	0
SSE %	10.0	10.30	10.7	10.79	11.2	10.3	9.78
HRE %	4.28	4.15	3.8	3.0	1.78	−5.02	−10.61
Comp. Var.	Low in.						
NSSE	0	0	0	0	0	0	0
SSE %	12.8	13.1	13.0	12.82	12.5	11.76	11.15
HRE %	5.8	5.51	4.22	2.43	1.26	−5.98	−11.82
Consumption							
NSSE	1.163	1.143	1.124	1.108	1.097	1.031	.9785
SSE	1.115	1.091	1.071	1.055	1.044	.9867	.9396
HRE	1.147	1.130	1.117	1.112	1.110	1.106	1.105

Table 11 provides some further results under a range of alternative settings for the temptation parameters. They are presented here to illustrate the sensitivity of results to the strength parameter (σ) and future discount parameter (φ) values.

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